Kansas v. Nebraska & Colorado

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KANSAS' EXPERT REPORT ON NEBRASKA'S 5-RUN PROPOSAL

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I. Background

The RRCA Groundwater Model (Model) is used to compute stream depletions caused by groundwater pumping in each of the three States and to compute an imported water supply credit to compensate Nebraska for the effects of seepage associated with water that originates from the Platte River system. Because the Model is not completely mathematically linear, the individual impacts associated with pumping in each of the States and with the imported water supply credit as computed using the procedures described in the Final Settlement Stipulation (FSS) will not always sum to the total impact that would be computed by the Model if all of the impacts due to pumping and seepage of imported water were to be considered simultaneously. This latter total impact has been referred to as the "virgin water supply metric" and was used by Kansas to evaluate Nebraska's 2007 proposal to change the method for computing impacts as specified in the FSS. At that time, Kansas criticized Nebraska's proposal because a cursory examination of the proposal indicated that results using the proposed method deviated from the metric to a greater degree than the method specified in the FSS.

In 2012, Nebraska returned to the method that it had proposed in 2007 after advocating an alternative 16-run proposal in the interim. Nebraska's current proposal is now referred to as the 5-Run Proposal. For clarity, the method prescribed in the FSS will be referred to as the RRCA Method. The Special Master has determined that the RRCA Method specified in the FSS includes consumption of imported water in the determination of the computed water supply and that this consumption is contrary to other provisions of the FSS. Nebraska and Colorado have mutually agreed that the 5-Run Proposal should replace the RRCA Method.

II. Kansas' Evaluation of Nebraska's 5-Run Proposal

In the time allotted, Kansas has attempted to evaluate whether the omission of Mound seepage from the baseline of Nebraska's 5-Run Proposal was appropriate. Kansas did so by specifically evaluating: (1) the sufficiency of the calibration of changes in water levels in the Mound as simulated by the Model to changes in water levels in the Mound as measured during the period from 2001 to 2010 and the causes for any discrepancies; and (2) whether it is possible to arrive at a solution that would meet the Special Master's criteria and at the same time (a) eliminate the residuals caused by the hydrologic nonlinearity of the Republican River Basin and (b) assign responsibility for those residuals appropriately.

A. Sufficiency of the Model's Calibration for Purposes of Nebraska's 5-Run Proposal

The 5-Run Proposal changes the baseline condition that is used to calculate groundwater consumptive beneficial use (GW CBCU) with the Model from the historical condition to a condition where seepage from imported water, referred to as "Mound seepage," has been eliminated. This change places greater emphasis on the impact of Mound seepage on the determination of depletions due to pumping. A draft report prepared for the State of Nebraska by McDonald-Morrissey in April, 2006 (McDonald-Morrissey, 2006) recognized this potential impact and initiated a study to evaluate water level conditions in the Mound area and their impact on stream depletions and accretions calculated by the Model. In a subsequent draft report dated January 10, 2007 (McDonald-Morrissey, 2007) that was made available to Kansas, McDonald-Morrissey supplemented their evaluation of the Mound area. This latter report, however, focused on evaluating why imported water supply credits had declined in recent years and did not study or evaluate the impact of high water levels in the Mound area on calculations of stream flow depletions as the 2006 report contemplated it would be.

Kansas has attempted to conduct such a study or evaluation within the time frame it has been allotted to assess the impact of changing the baseline condition from the historical condition to a condition without the effects of Mound seepage. The task involved comparing model results to measured groundwater level data as suggested by McDonald-Morrissey in their 2006 report (McDonald-Morrissey, 2006, page 7).

Kansas compiled available groundwater level data from two sources, the U. S. Geological Survey (USGS) and from the State of Nebraska. In response to discovery requests from Kansas, Nebraska directed Kansas to a website where the Nebraska groundwater level data could be downloaded. Data from the USGS is also available from an online database.

The compiled groundwater level data was compared to historical groundwater levels calculated by the RRCA Model. The purpose of this comparison was to evaluate the nature of overestimation or underestimation of measured groundwater levels and of temporal trends in groundwater levels. This evaluation focused on the period after 2000 because this period is beyond the calibration period that was considered as part of the calibration of the Model at the time of the settlement, is more representative of current hydrologic and institutional conditions, and reflects water use data provided by the States pursuant to the requirements of the FSS.

A comparison of average measured groundwater level elevations for the period 2001-2010 to equivalent Model results shows patterns of consistent overestimation and underestimation. Ideally, differences between Model results and measured groundwater levels would exhibit a random spatial pattern of overestimation and underestimation. When overestimation or underestimation becomes predominant over an area, it can be an indication of bias or error in model inputs. For example, McDonald-Morrissey, on page 7 of their April 2006 report (McDonald-Morrissey, 2006), observed that computed groundwater levels in the Mound area for several Nebraska counties were “consistently too high” and warranted further study.

Figure 1 shows a map of the Model domain and also shows the locations of wells where groundwater level elevation data are available for the period 2001 to 2010. The colors at each of the well locations indicate whether the Model results underestimate or overestimate the measured groundwater levels and depict the degree of overestimation or underestimation. The legend at the lower left corner of the map describes what the individual colors represent. Shades of blue have been used to represent locations where Model results overestimate measured levels and shades of yellow to red represent locations where Model results underestimate measured levels. The intensity of the colors was selected to be greater in relation to the amount of overestimation or underestimation.

As shown on Figure 1, patterns of overestimation are shown in the eastern part of the Nebraska portion of the Model domain. This is an area where Mound seepage is occurring and may be the same area that McDonald-Morrissey was concerned about in their April 2006 report. Other areas of overestimation are also evident in the northwestern part of the Nebraska portion of the Model domain and in parts of the Colorado portion.

While areas of consistent overestimation or underestimation of computed groundwater levels are an indication of potential Model input errors, comparisons of changes in groundwater levels over time are a better indicator of Model input reliability. These comparisons allow for an examination of the Model’s ability to track longer term changes associated with Model inputs such as Mound seepage and pumping. Failure of the Model to track these longer term changes can be an indication of bias in the estimated amounts of Mound seepage or pumping.

Figure 2 shows a map of changes in groundwater levels between 2001 and 2010 over the Model domain as measured in numerous wells. The map was constructed from well location data points where groundwater level data were available in both 2001 and 2010 and a change in the measured groundwater level over that period could be determined. The change in measured groundwater level for the individual data points were then contoured using standard computer software to develop the colored map shown on Figure 2. The legend in the lower right corner of the map shows what each of the map colors represent. Areas of groundwater level decline are shown in increasing amounts from yellow to pink to red. Red represents areas of groundwater level declines that exceed 10 feet over the period from 2001 to 2010. Similarly, dark blue represents areas where groundwater levels have increased by 10 feet or more over the period.

As shown on Figure 2, much of the area in the western and north-northwestern parts of the Nebraska portion of the Model domain experienced groundwater level declines over the period from 2001 to 2010. The western area in Nebraska is contiguous with a similar area in Colorado and these are both areas where more intense pumping occurs in both Nebraska and Colorado. These are areas where groundwater levels have been declining for several decades in response to the pumping. The north-northwestern area includes areas where Mound seepage occurs. The general area of Mound seepage extends from this area toward the southeast roughly parallel to and near the northern boundary of Model domain. Over most of this distance, the northern boundary of the Model domain is the Platte River.

Figure 3 shows changes in groundwater levels from 2001 to 2010 as computed by the Model. The same color representations were used on Figure 3 as was used on Figure 2 to allow for a direct visual comparison between the two figures. Two things are readily apparent from visually comparing the two figures. First, the Model results (Figure 3) show two areas of blue and dark blue in the northernmost part of the Model domain near the Platte River. According to the Model results, groundwater levels in these two areas are computed to have increased up to more than 20 feet during the period from 2001 to 2010. In these same areas, the measured groundwater levels (Figure 2) indicate a decline in groundwater levels between 2001 and 2010. Second, measured groundwater levels in the western part of the Nebraska portion of the Model domain and in adjacent parts of Colorado have declined more than 10 feet (red areas on Figure 2) over a much larger area than is shown by the comparable Model results (Figure 3).

The first type of discrepancy between Model results and the measured changes in groundwater levels that appears on Figures 2 and 3 is related to Mound seepage. Figure 4 shows the locations of Model cells where Mound seepage is assumed to occur in the Model domain and depicts the intensity of that seepage in the Model by color gradations. Figure 4a shows the average Mound seepage amounts for the period from 2001 to 2010. The legend in the lower right hand corner of Figure 4a describes the relationship between the colors and the amounts of Mound seepage.

Figure 4b shows the locations of Mound seepage superimposed on the map of computed changes in groundwater levels (Figure 3). The blue and dark blue areas on Figure 3 were noted above as areas where the Model results show a significant increase in groundwater levels from 2001 to 2010, whereas the measured data showed that groundwater levels actually decline. These areas correspond directly to areas of significant Mound seepage during the period from 2001 to 2010 as shown on Figure 4a.

This comparison indicates that Mound seepage is overestimated in these areas and that the overestimation of Mound seepage is the reason that Model calculations show a significant increase in groundwater levels from 2001 to 2010 while the measured groundwater levels show a decline.

The second type of discrepancy that appears on Figures 2 and 3 occurs in the Mound area, but also more broadly, and is most likely related to underestimated amounts of net irrigation pumping. Net irrigation pumping is the difference between the amount of irrigation pumping and the amount of groundwater return flow that is assumed to be associated with that pumping. As described above, the measured groundwater levels show a larger area of groundwater level declines exceeding 10 feet in western Nebraska and adjacent areas in Colorado (Figure 2) than the equivalent area as computed by the Model (Figure 3). To assess the extent of this behavior of the Model, scatter diagrams were prepared to directly compare measured changes in groundwater levels from 2001 to 2010 in wells to what the Model computed the change to be at the location of each of the wells.

First, scatter diagrams were prepared for wells within the geographic area of each State. This allows an evaluation of how well the Model is performing on a State by State basis. Figures 5, 6 and 7 show scatter diagrams for wells in Kansas, Nebraska, and Colorado, respectively. The scatter diagram for Kansas (Figure 5a) will be described generally to illustrate what is portrayed on each of the scatter diagrams. The horizontal axis of the diagram is the change in groundwater level for the period from 2001 to 2010 as measured in various wells. These measured values were the basis for the contour map shown on Figure 2. The vertical axis is the change in groundwater level from 2001 to 2010 as computed by the groundwater Model. The particular values are calculated by interpolating Model results for 2001 and 2010 at each well location and then subtracting the 2010 result from the 2001 result. A negative value indicates that the groundwater level in 2010 was lower than the value in 2001. In other words, a negative value indicates that the groundwater level was computed to have declined between 2001 and 2010. Conversely, a positive value indicates that the ground water level was computed to have risen between 2001 and 2010. The value of groundwater level change computed by the Model at a particular well location is then paired with the measured change in groundwater level between 2001 and 2010 for that same well as shown on Figure 2. If the Model had computed changes in groundwater levels that were exactly the same as the measured change in groundwater level, the point would plot along a 45-degree line on the diagram. The 45-degree line is shown as a bold dark line on the diagrams for easy reference.

The scatter diagram is a diagnostic tool that can be used to examine potential bias in Model results. Generally speaking, an unbiased result is one in which the data points plot along the 45-degree line on the diagram and are randomly distributed above and below the line. If a model was perfect, which models by definition are not, all of the data points would plot exactly along the 45-degree line. Furthermore, as long as the data points are equally distributed above and below the 45-degree line in a random pattern, the Model results would not exhibit bias. If the data points show a pattern but the pattern does not lie along the 45-degree line, this could be an indication of a structural problem with the model. Structural problems could include model input data or the characteristics of the Model itself.

Figure 5a shows the scatter diagram for measured groundwater level data from wells in Kansas. The data points are "scattered" about the 45-degree line in a random pattern. Furthermore, there is no visual predominance of points plotting above the line as opposed to below the line. Consequently, the Model results in Kansas do not indicate a bias or potential structural problem with the Model itself.

Figure 5b is a well series plot that displays the results in a slightly different fashion than the scatter diagram. Rather than plotting the computed result on one axis and the measured result on the other axis as the scatter diagram does, Figure 5b plots both values at a given point along the x-axis. On this figure, each position along the x-axis represents a particular well location. At each position, the computed result and the measured result are plotted with different symbols. The positions have been arranged so that the well with the lowest (in this case, most negative) measured result appears on the left, followed by the next higher (less negative) measured result to the right, and so on across the graph. The results shown on Figure 5b again show computed results plotting above and below the measured result in a random pattern.

Figure 6a shows the scatter diagram for measured groundwater level data from wells in Nebraska. The data points shown on this figure more frequently plot above the 45-degree line than below the line. This bias is particularly evident for the more negative values as seen on Figure 6b which is the well series plot for the Nebraska wells. Negative values represent declines in the groundwater levels between 2001 and 2010. The more negative the value the greater the amount of decline over the period. When a computed value is less negative than the corresponding measured value, it means that the Model is calculating a smaller groundwater level decline than the decline that was measured. Thus the data points on the left side of Figure 6b show that the Model is underestimating the measured groundwater level decline much more frequently than it is overestimating the measured decline.

Figure 7a shows the scatter diagram for measured groundwater level data from wells in Colorado. Figure 7b is the corresponding well series plot. The results depicted on Figures 7a and 7b show the same predominance of underestimating measured groundwater level declines as was seen in the plots for Nebraska.

The bias in results shown on Figures 6 and 7 are even more apparent when data are grouped geographically. For example, Figures 8a and 8b shown the scatter diagram and the well series plot when only wells in Yuma County, Colorado are considered. The scatter plot (Figure 8a) shows that relatively few points plot below the 45-degree line. The bias shown by the scatter plot is significant. As shown on Figure 8b for well locations where the measured groundwater level decline between 2001 and 2010 was more than about 8 to 10 feet, the Model result is always above the measured value indicating that the Model is always underestimating the groundwater level decline over this period of time.

The average measured groundwater level change for the wells shown on Figures 8a and 8b for the period 2001 to 2010 was about negative 10 feet. This means than on average for wells in Yuma County, groundwater levels declined almost 10 feet over the period. The average groundwater level change at these well locations over this period computed by the Model is about negative 5 feet. In other words, the Model is computing an average decline in groundwater levels over this period of just over 5 feet while the measured groundwater level data show an average decline of almost 10 feet. Thus, over this period from 2001 to 2010 the Model is only computing about one-half of the groundwater level decline that actually occurred.

The Model results for western counties in Nebraska show similar bias as that shown for Yuma County in Colorado. For example, Figures 9a and 9b show a scatter diagram and well series plot for well locations in Chase County, Nebraska. The same bias that was evident in the plots for Yuma County, Colorado is evident on these figures. The average measured decline in groundwater levels for wells in Chase County over the period from 2001 to 2010 was over 8 feet. The average decline for that period at those well locations computed by the groundwater Model was less than 6 feet. An evaluation of results for wells in Perkins and Dundy counties in Nebraska showed a similar bias. It is noteworthy that the bias is most acute at well locations where the largest groundwater level declines occurred over the period from 2001 to 2010.

Figures 10, 11 and 12 are groundwater level hydrographs for three well locations, one in Lincoln County, Nebraska (Figure 10), one in Perkins County, Nebraska (Figure 11) and one in Yuma County, Colorado (Figure 12). Each figure shows a hydrograph in the upper pane and a map of the particular well location in the lower pane. Each hydrograph shows three segmented lines. The blue segmented line depicts measurements of groundwater levels taken at different times over a multi-year period as shown on the x-axis. Each blue symbol represents one measurement. Each of the blue symbols has been connected by a straight line to facilitate viewing of the data. The red symbols and red lines depict groundwater levels computed by the groundwater Model at the same locations and times as the measured data. The difference between the red segmented line and the blue segmented line represents the difference in groundwater level elevation between the Model result and the measured value at the well location for each of the times when groundwater levels were measured. Since the bias described earlier dealt with the trends or changes over time in the Model results as opposed to the elevations, a green segmented line was plotted to shift the red segmented line into the range of the measured data (blue segmented line). By comparing the green segmented line to the blue segmented line, the difference in the trends or changes over time computed by the groundwater can be more easily compared to the trends or changes over time shown by the measured data.

The comparisons on Figures 10, 11 and 12 show that the trends in the results from the groundwater Model begin to depart from the trends in the measured data after about 2000. As shown by the red and green segmented lines on Figure 10, the Model results show a sharp increase in computed groundwater levels beginning in about 2007. The measured groundwater levels (blue segmented line) do not show this increase. Furthermore, the change in measured groundwater levels between 2001 and 2010 show a decline of about 10 feet. The change in groundwater level computed by the Model over this period is an increase of about 15 feet. This example hydrograph illustrates the nature of the bias in the area of Mound seepage noted on the comparison of Figures 2 and 3 that was discussed previously.

On Figures 11 and 12, the measured groundwater levels in these wells as shown by the blue segmented line continue to decline after 2000, and the rate of decline appears to increase after 2000. Model results (red and green segmented lines), on the other hand, tend to show less decline and the rate of decline appears to be decreasing. These example hydrographs illustrate the nature of the bias that was noted previously on the comparison of Figures 2 and 3 in the areas of pumping in western Nebraska and eastern Colorado and further defined by the scatter diagrams and well series plots shown on Figures 6 through 9.

The results shown on Figures 6 through 12 for wells in western Nebraska and eastern Colorado indicate that net irrigation pumping may be underestimated in the Model in parts of Nebraska and Colorado. This underestimation may be related to the estimated amounts of return flow associated with irrigation pumping reported by Nebraska and Colorado. Table 1 shows the amounts of return flow associated with irrigation pumping that has been reported by each State for input to the groundwater Model. Also shown on the table are the reported amounts of irrigation pumping and the fraction or percentage of irrigation pumping that is represented by the return flow associated with the pumping.

Figure 13 is a graph that depicts how the return flow fractions or percentages have changed over time. As shown by the values on the table and by the figure, the return flow fraction or percentage has decreased for all of the States over time, corresponding to the use of more efficient irrigation practices over time. The shift from flood irrigation practices to center pivot irrigation systems was likely responsible for much of the increase in efficiency. In addition, irrigation return flows are affected by water management practices. With continuing declines in groundwater levels and potential declines in well yields along with the imposition of more stringent water allocations, irrigators will likely respond with more efficient application of water to meet crop demands, thereby reducing irrigation return flows.

As shown on Figure 13, the fraction or percentage of return flow from irrigation in Kansas over the past 10 to 20 years has continued to decrease to levels between 10 and 15 percent. The increased use of center pivot irrigation systems as well as other system modifications such as low pressure drop nozzles have allowed efficiencies to approach 90 percent and have reduced the fraction of return flow to levels approaching 10 percent. These continued increases in irrigation efficiency and reduced irrigation return flow seen in Kansas are not evident in the reported values for Nebraska and Colorado. As shown on Figure 13, the fraction of irrigation return flow reported by Nebraska has remained at just over 20 percent since 2000. Similarly, the fraction of irrigation return flow reported by Colorado has a slight downward trend over the past 10 to 20 years but remains in the general range of 17 to 18 percent.

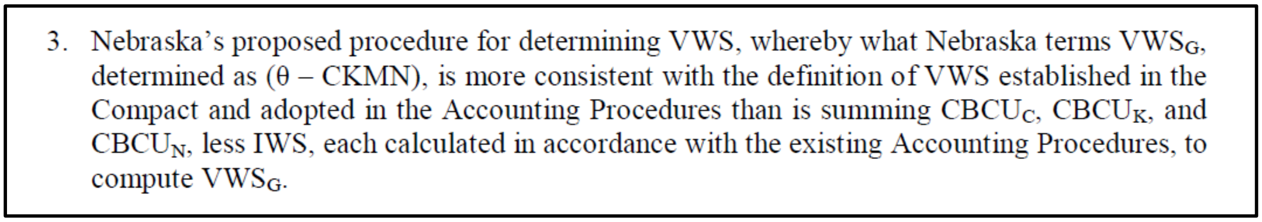
Table 2 shows the differences between the percentages of return flow reported by Colorado and Nebraska from the percentages reported by Kansas since 2000. The table also shows the amount of return flow that these differences in percentage represent for Colorado and Nebraska since 2000. For Colorado, this means that net irrigation pumping could be underestimated by 22,000 to 34,000 acre feet per year over these years if irrigation practices in Colorado were actually as efficient as those in Kansas. Similarly, for Nebraska it means that net irrigation pumping could be underestimated by 90,000 to 158,000 acre feet per year over these years if irrigation practices in Nebraska were actually as efficient as those in Kansas. Results from the groundwater Model for changes in groundwater levels from 2001 to 2010 for well locations in Kansas showed no apparent bias when compared to measured changes in groundwater levels (Figures 5a and 5b). Consequently, the most likely explanation for why groundwater Model results for both Colorado and Nebraska show a bias toward underestimation of declines for that period is that irrigation return flows have been overestimated and net irrigation pumping has been underestimated.

B. A More Appropriate Remedy: The Integrated Solution

Although the 5-Run Proposal does not include seepage from imported water when calculating the stream depletions caused by each State's pumping, the sum of the stream depletions or accretions caused by each State's pumping and the imported water supply credit do not equal the total impact of calculating the effects of pumping in each State and seepage of imported water simultaneously. In the discussions below, the sum of the stream depletions caused by each State's pumping and the imported water supply credit calculated separately will be referred to as the "sum of the impacts". The total impact of calculating the effects of pumping in each State and seepage of imported water simultaneously will be referred to as the "total impact".

As discussed above, Kansas criticized Nebraska's 2007 proposal at that time because it departed further from the "VWS metric" than the RRCA Method. The problem that Kansas recognized at that time continues to be a problem with the 5-Run Proposal. Kansas has made evaluations that show that as pumping continues and depletions to stream base flows continue to increase, the departure between the sum of the impacts and the total impact will likely increase over time.

Prior to reaching agreement with Colorado regarding the 5-Run Proposal, Nebraska was advocating a 16-run proposal. One of the features of the 16-run proposal was that the sum of the impacts did equal the total impact. In other words, the 16-run proposal met the "VWS metric". Colorado and Kansas did not support the 16-run proposal for various reasons that are described in expert reports submitted in arbitration proceedings and initially in this matter before the Special Master. In the arbitration proceedings before Karl J. Dreher, the 16-run proposal was presented and evaluated. While the arbitrator did not accept the 16-run proposal, he did make the following conclusion regarding the "VWS metric" that was the foundation for the 16-run proposal.



3. Nebraska's proposed procedure for determining VWS, whereby what Nebraska terms VWS_G, determined as $(\theta - CKMN)$, is more consistent with the definition of VWS established in the Compact and adopted in the Accounting Procedures than is summing $CBCU_C$, $CBCU_K$, and $CBCU_N$, less IWS, each calculated in accordance with the existing Accounting Procedures, to compute VWS_G.

From Arbitrator's Final Decision, June 2009, page 61.

The Arbitrator also concluded that under the 16-run proposal, the IWS credit would generally be greater than the credit determined using the RRCA Method. Under the 5-Run Proposal currently proposed by Nebraska and supported by Colorado, the IWS credit would be calculated using the same procedure as that used in the RRCA Method. Thus the 5-Run Proposal would not generally increase the IWS as would have occurred under the 16-run proposal. However, the 5-Run Proposal does not produce a result where the sum of the impacts is equal to the total impact. As will be shown below, the departure of the sum of the impacts from the total impact is expected to increase in the future, and the failure of the 5-proposal to meet this condition has a negative impact on Kansas.

Although Arbitrator Karl J. Dreher did not accept the 16-run proposal, he recommended that the States reconvene the Technical Groundwater Modeling Committee to "thoroughly re-evaluate the non-linear response of the Model when simulated stream drying occurs" and "to re-evaluate the existing procedures for determining CBCU and IWS" (Arbitrator's Final Decision, June 2009, page 71, number 2). Among the reasons that the arbitrator did not accept the 16-run proposal was the fact that the residual or difference between the sum of the impacts and the total impact would essentially be divided among two States without consideration of other factors such as groundwater storage (Arbitrator's Final Decision, June 2009, page 13, number 30). While the Arbitrator recognized the total impact defined by calculating the impact from each State's pumping and IWS seepage simultaneously was an estimate and should not be viewed as a "true" value as was suggested by Nebraska (Arbitrator's Final Decision, June 2009, page 7, number 16), he concluded that it was more consistent with the definition of VWS established in the Compact and adopted in the Accounting Procedures (Arbitrator's Final Decision, June 2009, page 61, number 3).

Given the determinations made by the Special Master in this matter and considering findings of the Arbitrator described above, Kansas has evaluated the 5-Run Proposal in an effort to find a method for computing GW CBCU that would adhere to the determinations by the Special Master and would give a result in which the sum of the impacts would equal or nearly equal the total impact. A method developed by Kansas that would achieve these goals is described below and will be referred to as the Integrated Solution. Subsequently, comparisons between results obtained using the Integrated Solution discussed below and both the RRCA Method and the 5-Run Proposal will be presented and discussed.

Differences between the sum of the impacts and the total impact are related to the non-linear response of the groundwater Model under certain hydrologic conditions. These conditions are primarily associated with what has been referred to as "stream drying". The occurrence of stream drying is variable and can be related to both variations in groundwater pumping and variations in recharge from precipitation. Depletions to stream flow in the future associated with ongoing and historical pumping can be expected to continue to increase. This continuing increase in depletions will increase the prevalence of stream drying and the differences between the sum of the impacts and the total impact will continue to increase.

A preferable alternative approach to addressing the effects of stream drying on the differences between the sum of the impacts and the total impact is to evaluate depletions incrementally over a range of pumping conditions from no pumping to the full amount of pumping at any particular time period. In other words, the approach would be to integrate the depletions with respect to the amount of pumping from no pumping up to the full amount of pumping. The process of integration is a fundamental concept in mathematics and calculus. In practical terms, integration describes the process of determining the total amount of something (a function) that varies over a range of conditions (a variable). In our case, we want to determine the total amount of impact to stream flow associated with a range of pumping conditions. We also want to determine how much each State's pumping contributes to that total impact since each State's pumping will impact stream flows differently. By formulating the total impact as a sum of the partial impacts attributable to each State and integrating over the range of pumping, the portion of each State's contribution to the total impact can be determined.

The integration of partial impacts described above must be evaluated numerically since we do not have a simple function that describes each State's relative capacity to impact stream flows. This means that the integration is accomplished over a series of discrete intervals or increments. The intervals or increments span the range of pumping from no pumping to the total amount of pumping. If the increment of the integration process is made small enough, the resulting estimates of impacts to stream flow or depletions due to pumping in each State will sum to a value that is equal to the total impact. This approach is referred to as the "Integrated Solution" and was developed by Dr. Sam Perkins of the Kansas Department of Water Resources (KDWR). The Integrated Solution is described in more detail in Appendix B to this report.

As described above, the Integrated Solution can be formulated to produce a result where the sum of the impacts is equal to the total impact. In short, the increment of the integration process can be adjusted downward until the sum of the impacts is very close to the result of computing the total impact of pumping and seepage from imported water simultaneously. For purposes of the comparisons to be discussed below, an increment size was selected that produced results that sum to within less than one percent of the total impact.

The Integrated Solution as described herein provides a method that achieves the goals of satisfying the Special Master's determination that GW CBCU must be computed without the inclusion of seepage from imported water and of satisfying the arbitrator's finding that having the sum of the impacts equal the total impact would be more consistent with the definition of VWS in the Compact and the FSS. This latter goal eliminates residual depletions associated with stream drying that would otherwise not be included in the determination of GW CBCU. This residual GW CBCU would then be included in the calculations of the computed water supply, allocations to the States and determinations of compact compliance.

In the following sections, comparisons are presented to illustrate how results using the RRCA Method and the proposed 5-Run Proposal are different from results obtained from the Integrated Solution. Specifically, the comparisons will be presented for the difference in GW CBCU, the difference in the computed water supply (CWS), the difference in the allocation of that computed supply to the States, and the difference in the compliance balance for each State associated with differences in GW CBCU and allocation.

The comparisons were made over a study period extending about 51 years into the future. Two different assumptions were made regarding climatic conditions for the study period. In the first case, an average climatic condition over the study period was assumed to occur in each year of the study period. This condition was the same condition that was developed by Kansas for purposes of illustrating potential future effects of pumping in previous submissions (see Fig. 7, App. C to Kansas' Petition, May 2010) in this case. It is also the same condition that was used by the States of Nebraska and Colorado to illustrate potential future conditions in their prior expert work submitted in this case (see for example Schreüder Report, June 19, 2012).

In the second case, a variable climatic condition was assumed for the study period. This variable climatic condition was the same condition that has been used by Kansas in prior expert work related to the arbitration in 2009 (see Larson and Perkins, January 20, 2009). In summary, the variable climatic condition uses historical climatic data for the years 1990 to 2006, a 17-year period. The sequence of climatic conditions over the 17-year period was repeated three times to create a 51 year study period.

1. Effects on Calculation of GW CBCU

The calculations of GW CBCU using the 5-Run Proposal by Nebraska and using the RRCA Method both produce results where the sum of the impacts may not equal to the total impact. The reason that Kansas did not accept the 5-Run Proposal in 2007 was that the sum of the impacts departed from the total impact to a greater degree than would occur using the method prescribed in the FSS. The departure between the sum of the impacts and the total impact using both the 5-Run Proposal and the RRCA Method can be expected to increase as stream drying becomes more acute in the future.

To illustrate this expectation, Figures 14 and 15 have been prepared to illustrate how the residual GW CBCU could be affected in the future under the RRCA Method and under the 5-Run Proposal. Figures 14a and 14b show differences in GW CBCU between results calculated using the 5-Run Proposal and the RRCA Method and results computed using the Integrated Solution. The differences were computed over the study period assuming the average future climatic condition and the variable climatic condition described previously. The integration interval used in applying the Integrated Solution was selected so the sum of the impacts would be very close to the total impact. Consequently, differences from results using the Integrated Solution are, in effect, differences from the total impact.

A positive value on Figures 14a and 14b indicates that GW CBCU calculated using either the RRCA Method or the 5-Run Proposal would be larger than it would be using the Integrated Solution. In other words, a positive value would indicate that GW CBCU is being overestimated relative to the Integrated Solution. Since the Integrated Solution effectively makes the sum of the impacts equal the total impact, departures from the Integrated Solution are analogous to residual depletions cited by Arbitrator Dreher (Arbitrator's Final Decision, June 2009, page 12, number 27, for example). Conversely, a negative value indicates that either the RRCA Method or the 5-Run Proposal would underestimate GW CBCU relative to the integrated method.

Figures 14a and 14b show an increasing departure over the study period of GW CBCU calculated using the RRCA Method and using the 5-Run Proposal as opposed to the Integrated Solution. This means that under both procedures, residual depletions can be expected to increase over time under the average climatic condition used in the analysis. In his testimony to the Special Master, Dr. Schreüder acknowledged that residual depletions are real and that during the historical period from 1981 to 2006, the residuals associated with the RRCA Method went both ways, "Sometimes we overestimate; sometimes we underestimate" (Transcript of Proceedings, August 2012, page 743). Dr. Schreüder's characterization will be discussed further below. However, as stream drying becomes more acute in the future, one can expect that the residuals will be persistently one way, either overestimating or underestimating, as shown by the results on Figures 14a and 14b.

It is also worth noting that the RRCA Method was based, at least in part, on the adoption of the groundwater Model. The process of adopting the groundwater Model included recognition that certain agreed upon inputs to the Model might overestimate or underestimate depletions from pumping. For example, the extent of increased precipitation recharge on irrigated land was extensively discussed and a compromise agreement reached for purposes of settlement (Final Report of the Special Master, 2003, page 20). If the agreed upon increase in precipitation recharge was overstated, net pumping on the irrigated land would effectively be underestimated and GW CBCU associated with that pumping would also be underestimated. Similarly, groundwater recharge derived from irrigation pumping in Nebraska was assumed to decrease from 30% in 1960 to 20% in 2000 associated with an assumed increase in efficiency from70% in 1960 to 80% in 2000 (Final Report of the Special Master, 2003, page 22). However, since 2000, Nebraska has continued to assume an efficiency of 80% and groundwater recharge of 20% from irrigation pumping. If actual irrigation efficiencies have continued to increase since 2000, groundwater recharge from irrigation pumping in Nebraska would be overstated and net pumping would be understated leading to GW CBCU being underestimated. This issue was discussed in previous sections of this report, but it shows that a tendency for the RRCA Method to overestimate GW CBCU could be partially offset by other factors that would lead to underestimation of GW CBCU.

The same trends in overestimation and underestimation of GW CBCU shown on Figures 14a and 14b can be expected under a variable climatic condition as shown by the results on Figures 15a and 15b. The differences shown on Figures 14a and 14b follow a smooth curved trend going into the future whereas the differences shown on Figures 15a and 15b are more variable. This variability is associated with year to year variations in climatic conditions that were assumed over the study period. Despite the variable nature of the results, the overall trends of the results shown on Figures 15a and 15b are similar to those shown on Figures 14a and 14b. This demonstrates that the trends in overestimation and underestimation of GW CBCU can be expected to persist regardless of variations in future climatic conditions.

2. Effects on Calculations of Computed Water Supply

The differences in GW CBCU described above will produce commensurate differences in the computed water supply (CWS) that is calculated using the accounting procedures. Figures 16 and 17 have been prepared to illustrate how the CWS would be different under the RRCA Method or the 5-Run Proposal as compared the value calculated using the Integrated Solution where the sum of the impacts would essentially equal the total impact.

As shown on Figure 16, the differences between results using the RRCA Method and using the Integrated Solution are positive and are expected to increase in the future under the average climatic scenario. The differences between results using the 5-Run Proposal and using the Integrated Solution are negative and are also expected to increase (become more negative) in the future under the average climatic scenario. Positive differences indicate that the computed water supply is being overestimated relative to the Integrated Solution. Similarly, negative values indicate that the computed water supply is being underestimated relative to the Integrated Solution.

Figure 16 shows that the degree of overestimation in the CWS using the RRCA Method is less than the degree of underestimation using the 5-Run Proposal. Also, the figure shows that the degree of overestimation using the RRCA Method tends to plateau in the future under the average climatic scenario. The degree of underestimation using the 5-Run Proposal, however, continues to increase over the study period. By the end of the study period, the amount of overestimation using the RRCA Method is about 10,000 acre feet per year and the amount of underestimation using the 5-Run Proposal is about 24,000 acre feet per year.

In is worth noting that the difference between the RRCA Method and the 5-Run Proposal at the end of the study period (2059) is about 34,000 acre-feet per year (from

10,000 acre-feet per year overestimated to 24,000 acre-feet per year underestimated). This difference is comparable to what was reported by Dr. Schreüder in his June 19, 2012 submittal in response to a request from the Special Master (Schreüder, June 19, 2012). For example, on the next to last page of that submittal, Dr. Schreüder shows results of his calculations for the year 2059, the last year of his study period. Under the column labeled "Change in CWS", Dr. Schreüder shows a total value of -35,464 acre feet. This represents the difference in the computed water supply in going from the RRCA Method to the 5-Run Proposal. However, as shown by the results on Figure 16, this difference is a result of overestimation of the CWS using the RRCA Method and underestimation of the CWS using the 5-Run Proposal as compared to the Integrated Solution. Furthermore, the degree of underestimation using the 5-Run Proposal is more than twice the degree of overestimation using the 5-Run Proposal.

Figure 17 shows the same type of comparison as Figure 16 except that the variable climatic scenario described previously was assumed rather than the average climatic scenario. As shown on Figure 17, the differences vary from year to year as a result of the varying climatic conditions that were assumed in the analysis as opposed to the relatively smooth lines shown on Figure 16 where an average climatic condition was assumed to occur year after year.

In spite of the varying differences associated with the assumed variations in climatic conditions, the trends shown on Figure 17 are quite similar to the trends seen on Figure 16. Again, the overestimation using the RRCA Method is expected to increase to something on the order of 10,000 acre feet per year but tends to stop increasing near the end of the study period. The underestimation using the 5-Run Proposal, on the other hand, increases over the study period and reaches a level of about 25,000 to 30,000 acre feet per year near the end of the period.

The results depicted on Figures 16 and 17 demonstrate that as stream drying becomes more acute in the future, one can expect that the sum of the impacts using either the RRCA Method or the 5-Run Proposal will become increasingly different from the total impacts. Further, one can expect the 5-Run Proposal to underestimate the CWS relative to what would be calculated using the Integrated Solution where the sum of the impacts would essentially equal the total impact and that the amount of underestimation could increase to as much as 25,000 to 30,000 acre feet per year over a 50-year period.

If the 5-Run Proposal had been implemented rather than the RRCA Method, the CWS and thus the estimated virgin water supply used in the accounting procedures would have been significantly underestimated. In response to questions from the Special Master regarding the impact of not assigning residual depletions on the total estimated virgin water supply, Dr. Schreüder indicated that the "currently approved procedure" or the RRCA Method "goes both ways. Sometimes we overestimate; sometimes we underestimate." (Transcript of Proceedings, August 2012, page 743) Dr. Schreüder went on to indicate that the average impact to Kansas allocation for the period from 1981 to 2006 was a small negative number (page 744). As best he could recall the number was -136 acre feet over the period but that the number was in his report (page 744).

While the number cited by Dr. Schreüder is not apparent from the various reports he has submitted, Dr. Schreüder's characterization that the RRCA Method tended to both overestimate and underestimate the CWS during the period from 1981 to 2006 appears to be accurate. What Dr. Schreüder did not indicate was what the 5-Run Proposal would have shown for the period from 1981 to 2006. If the 5-Run Proposal had been used to compute the CWS for this period, it would have consistently underestimated the CWS relative to the Integrated Solution by an average of over 7,000 acre feet per year. Furthermore, most of this underestimation in the CWS would have translated to reduced allocations to Nebraska and Kansas. The pattern of consistent underestimation of CWS (and consequently consistent under estimation of the total estimated virgin water supply) that would have occurred historically if the 5-Run Proposal had been used rather than the RRCA Method can be expected to continue and become more pronounced in the future as shown by Figures 16 and 17.

3. Effects on Calculation of Allocations to the States

The CWS is used in the accounting procedures as part of the determination of how much of the computed virgin water supply is allocated to each State. These allocations are then compared with computed CBCU and ultimately used to determine compact compliance. Figures 18 and 19 were prepared to illustrate how differences in the CWS described previously would affect the amount of water allocated to each State.

Figures 18a and 18b show how the allocations to each State would be different using the RRCA Method (Figure 18a) or the 5-Run Proposal (Figure 18b) as compared with the Integrated Solution. Results shown on Figure 18 are based on the average climatic scenario and the consequent differences in CWS shown on Figure 16. A positive result on Figure 18 indicates that a State would be allocated a greater amount than it would be allocated using the Integrated Solution where the sum of the impacts would essentially equal the total impact. Conversely, a negative value indicates that a State would be allocated a smaller amount than it would be allocated using the Integrated Solution.

Figure 18a shows that the RRCA Method would allocate more to Kansas and Nebraska and less to Colorado as compared with the Integrated Solution over the study period. This means that under the Integrated Solution Kansas and Nebraska would get less allocation over the years than they would receive under the RRCA Method given the average climatic conditions that were assumed in these calculations. Conversely, Colorado would get slightly more allocation over the years than it would have received under the RRCA Method.

Figure 18b shows that the 5-Run Proposal would allocate less to all the States as compared to the Integrated Solution. This means that under the Integrated Solution all of the States would get more allocation than they would have receive under the 5-Run Proposal given the average climatic conditions that were assumed in these calculations. As shown on the figure, Nebraska would receive the greatest increase in allocation and Colorado would receive the least amount. Kansas would receive an increased amount that is generally midway between the increase for Nebraska and Colorado.

Figures 19a and 19b show the same type of results as was shown on Figures 18a and 18b using the variable climatic scenario described previously. The overall trends in these results are similar to those shown on Figure 18 except that the amounts vary from year to year as influenced by the variable climatic condition as opposed to the average climatic condition that was used for the calculations shown on Figure 18.

4. Effects on Calculation of State Compliance Balances

The differences in GW CBCU, CWS and allocation described above combine to create a difference in the compact compliance balance for each State. For example, if GW CBCU increases, the CWS will be larger and the allocation of the CWS among the States will increase the allocations among the States. The increase in the allocation can be compared to the increase in GW CBCU to determine how each State's compliance balance is impacted. These comparisons of differences in CWS, allocation, and compact compliance balance and are the same types of comparisons that were presented by Dr. Schreüder in his June 19, 2012 report to the Special Master.

Figures 20a and 20b shows how the compact compliance balance for each State would be different considering the differences in GW CBCU, CWS and allocation that were shown on the previous figures. The results shown on these figures were calculated using average climatic conditions for future years. A positive value on these figures means that a State's compact compliance balance would be improved as compared to a result obtained from the Integrated Solution. For example, on Figure 20a, the compact compliance balance for Colorado and Kansas is larger when calculated using the RRCA Method than it would be using the Integrated Solution. Conversely, for Nebraska, the compact compliance balance is smaller when calculated using the RRCA Method than it would be using the integrated method.

Figure 20b shows that Colorado would retain a larger compact compliance balance under the 5-Run Proposal as compared to what would be calculated using the Integrated Solution. For Nebraska and Kansas, the 5-Run Proposal also shows a smaller compact compliance balance than what would be calculated under the Integrated Solution. In other words, the 5-Run Proposal has a negative impact on Kansas and Nebraska and a positive impact on Colorado as compared to the Integrated Solution.

Figures 21a and 21b depict the same type of results as Figures 20a and 20b except the results were calculated using variable climatic conditions for future years. The results shown on Figures 21a and 21b are similar to the results shown on Figures 20a and 20b except that the results are more variable from year to year associated with the year to year variations in the climatic conditions. Under the average climatic conditions, the results for future years follow a relatively smooth curved line whereas the results using the variable climatic condition do not follow a smooth curved line. The overall trend in results using the variable climatic condition is similar to the results using the average climatic condition.

In summary, Colorado receives a benefit to its Compact compliance under either the RRCA Method or the 5-Run Proposal relative to the Integrated Solution. The amount of benefit is approximately the same for the two methods. Kansas receives a benefit under the RRCA Method but is negatively impacted under the 5-Run Proposal relative to the Integrated Solution. Nebraska is negatively impacted under both the RRCA Method and the 5-Run Proposal relative to the Integrated Solution although the amount of negative impact is reduced under the 5-Run Proposal.

III. Conclusions

1. The 5-Run Proposal uses a baseline for determining pumping impacts by each State that does not include Mound seepage. Groundwater level data show that in areas where significant Mound seepage is estimated to occur, Model results for recent years show a bias that indicates Mound seepage rates are overestimated.

2. Groundwater level data for recent years also show a bias in Model results, both in the Mound Area and in western Nebraska and eastern Colorado, that is symptomatic of underestimated net irrigation pumping that, in turn, is likely related to an overestimation of the fraction of irrigation pumping that is assumed for estimating return flow from irrigation pumping.

3. The biases in the estimated amounts of Mound seepage and irrigation return flow indicated by groundwater level data collected from 2001 through 2010

must be resolved in order to reliably determine a baseline condition without Mound seepage, the amount of imported water supply credit, the amount of GW CBCU assigned to the States, the computed water supply and the allocations to the States.

4. An Integrated Solution for determining impacts to stream flow caused by pumping in each of the States is presented that is consistent with the Special Master's determination that the RRCA Method includes consumption of imported water by including Mound seepage in runs of the Model that are used to compute each State's GW CBCU.

5. The Integrated Solution is a mathematical approach for determining each State's impact on stream flow from pumping where the sum of each State's impact would be very nearly equal to the total impact computed by considering all of the pumping simultaneously.

6. The Integrated Solution is also consistent with the arbitrator Karl Dreher's finding that a method where the total impact of all pumping considered simultaneously would equal the sum of each State's impact would be more consistent with the definition of virgin water supply established in the Compact and adopted in the Accounting Procedures in the FSS.

7. The 5-Run Proposal advocated by Colorado and Nebraska is not a method where the total impact of all pumping considered simultaneously would equal the sum of each State's impact and would not be consistent with arbitrator Dreher's finding in this regard.

8. The 5-Run Proposal underestimates the computed water supply as compared with the Integrated Solution and the degree of underestimation is likely to increase in the future as stream drying conditions within the Model become more acute in the future due the effect of historical and ongoing pumping for irrigation.

9. Evaluations of the 5-Run Proposal assuming both an average future climatic condition and a variable future climatic condition show that underestimation of the computed water supply as compared with the Integrated Solution could increase to between 20,000 and 30,000 acre-feet per year over the next 50 years.

10. Residual GW CBCU as described by arbitrator Dreher would occur under the 5-Run Proposal and is also likely to increase in the future as stream drying becomes more acute. The Integrated Solution does not have a residual GW CBCU.

11. Based on the foregoing analysis and conclusions it is apparent that Nebraska's 5-Run Proposal is not an appropriate technical modification to the RRCA Accounting Procedures.

IV. References

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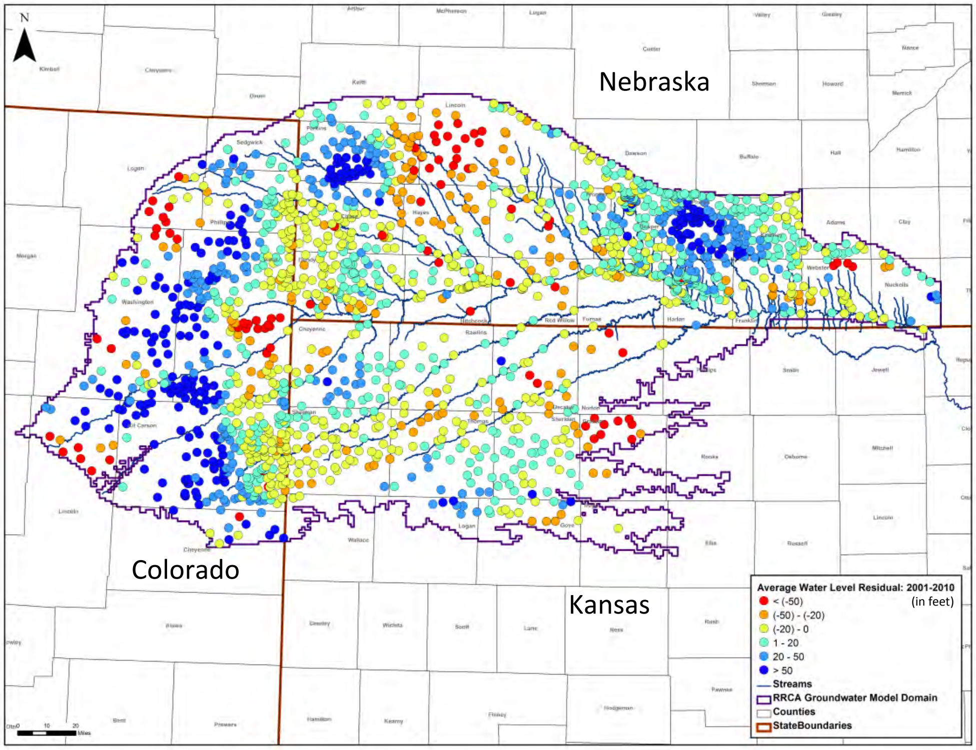


Figure 1: Map of RRCA Model Domain with Water Level Residuals for 2001-2010.

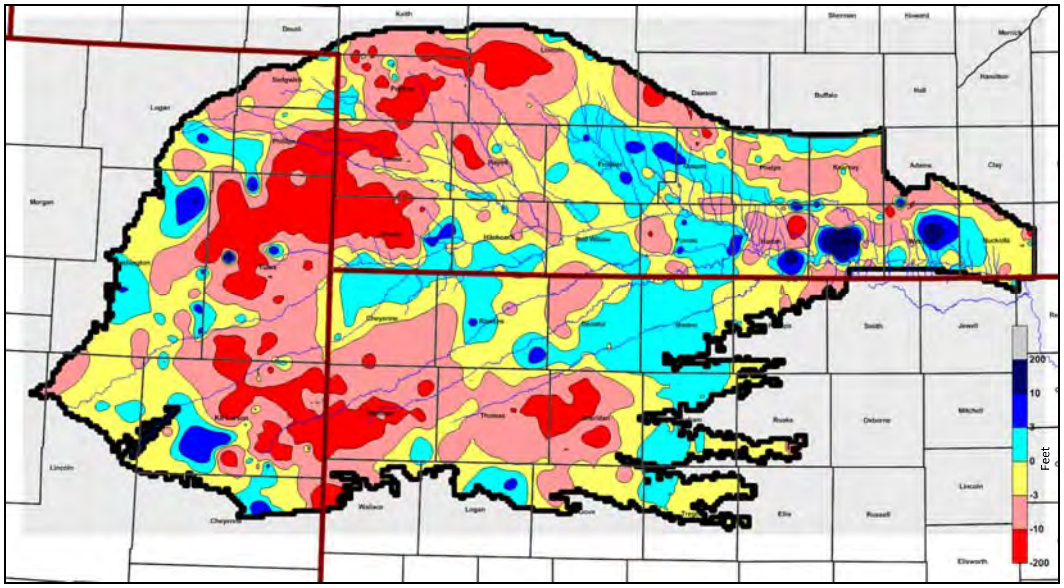


Figure 2: Map of Measured Changes in Groundwater Levels from 2001 to 2010.

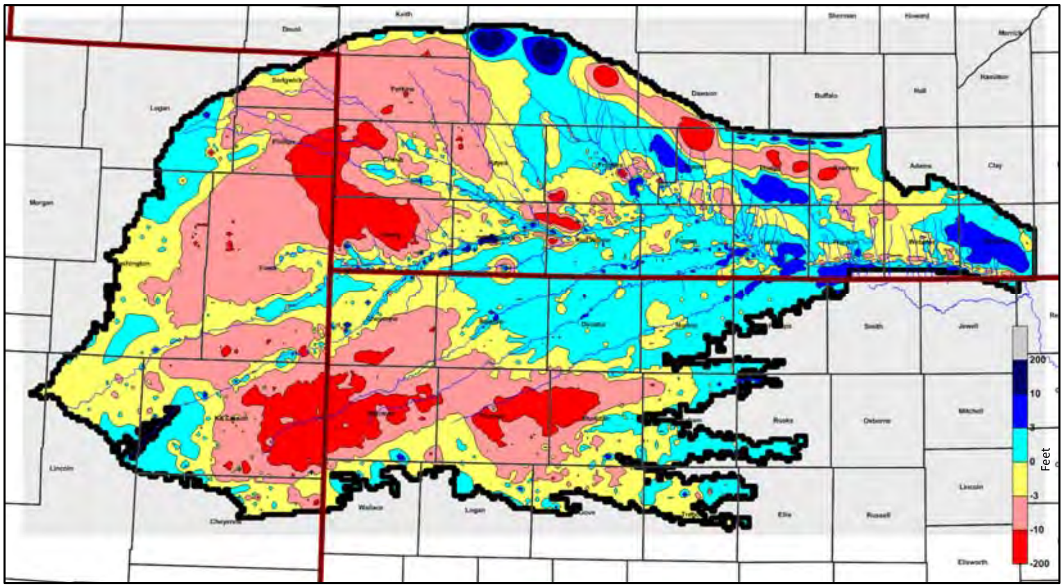


Figure 3: Map of Computed Changes Groundwater Levels from 2001 to 2010.

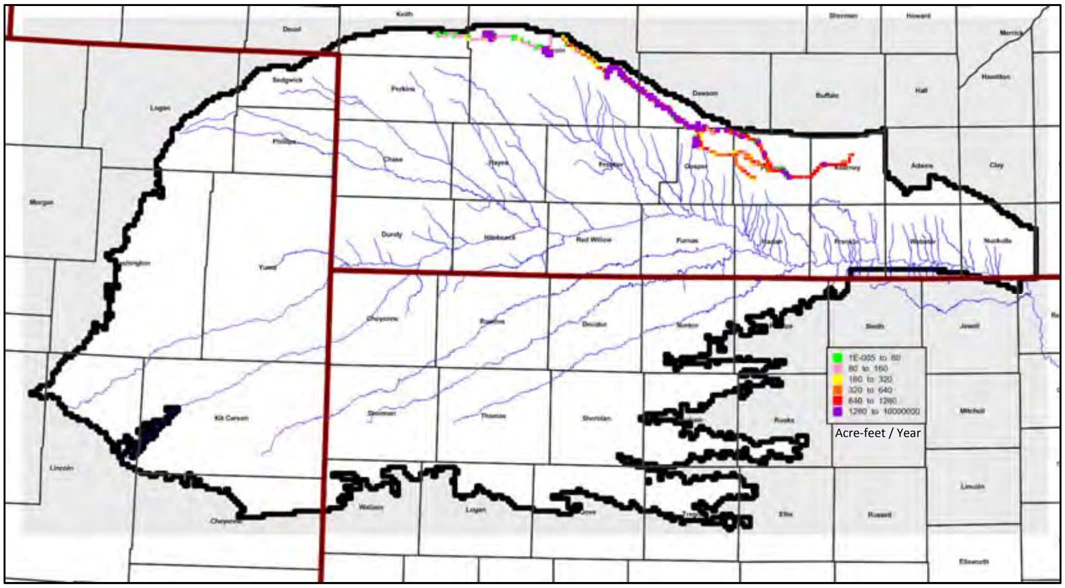


Figure 4a: Map Showing Locations and Amounts of Average Mound Seepage from 2001 to 2010.

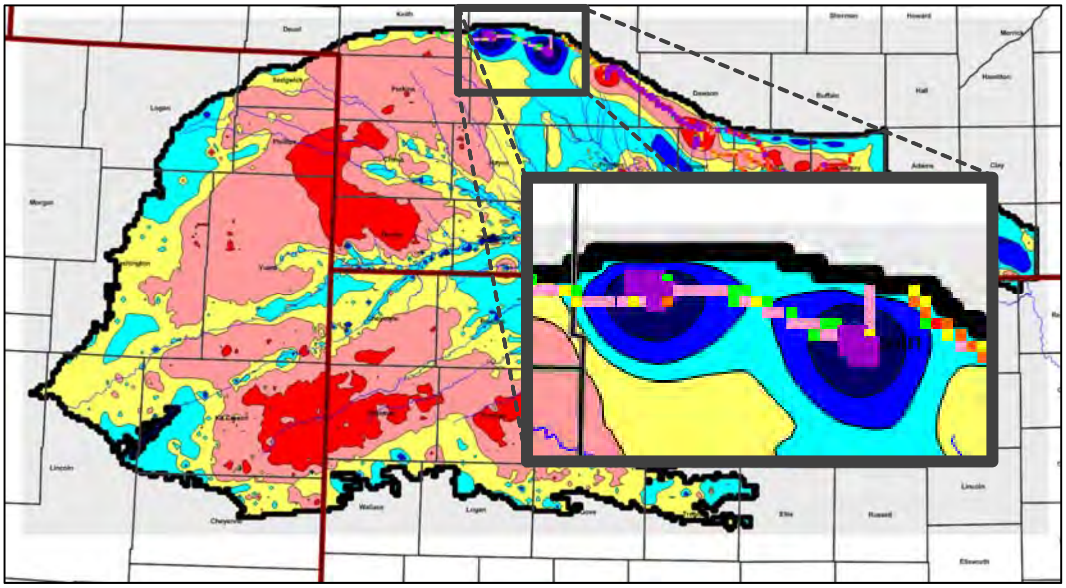


Figure 4b: Map Overlaying Locations and Amounts of Average Mound Seepage for 2001-2010 and Computed Groundwater Level Changes from 2001 to 2010.

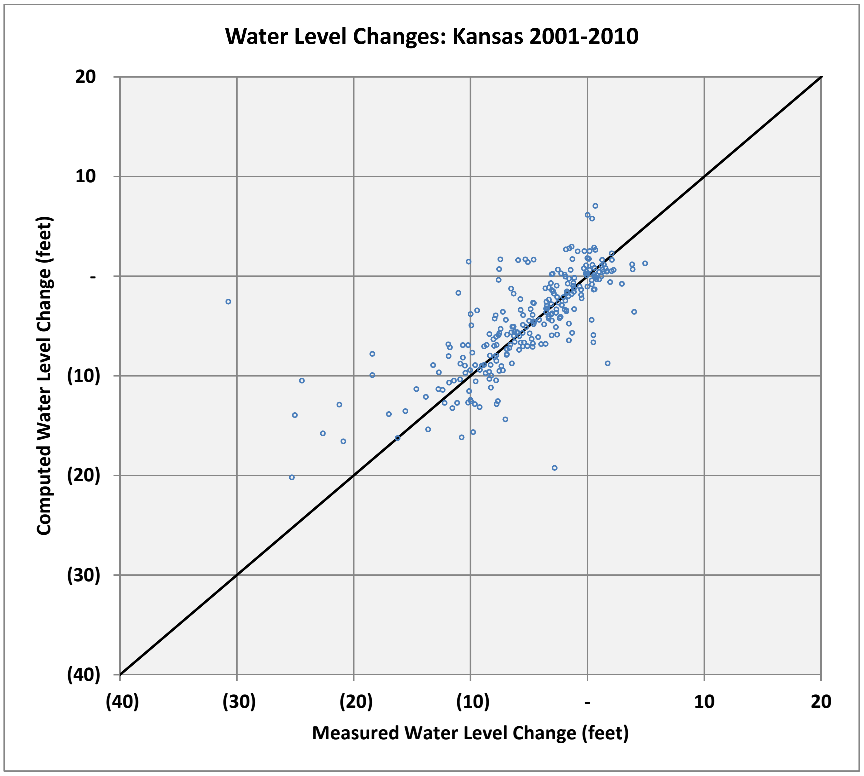


Figure 5a: Scatter Diagram of Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Kansas.

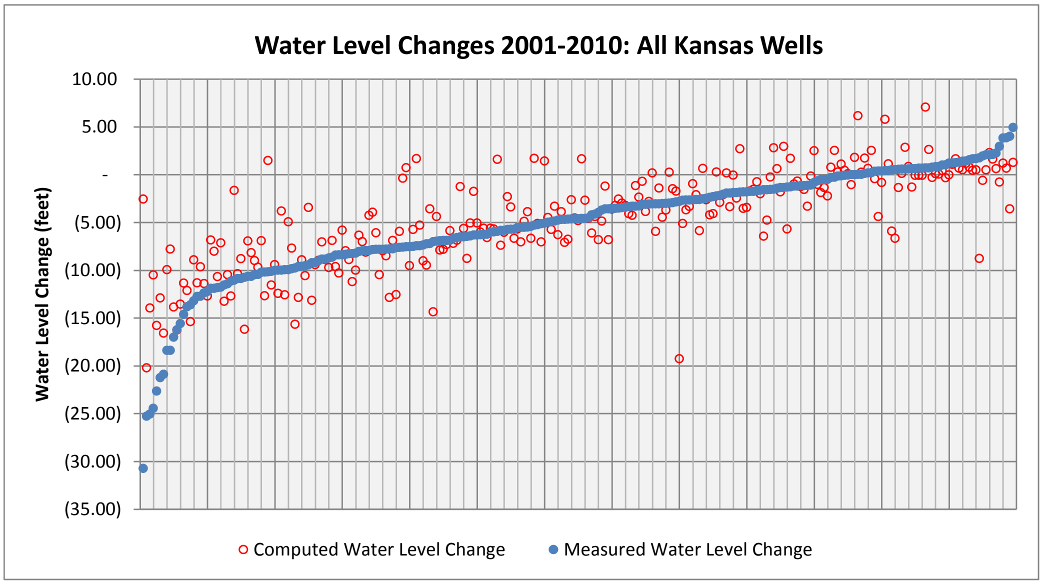


Figure 5b: Well Series Plot Comparing Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Kansas.

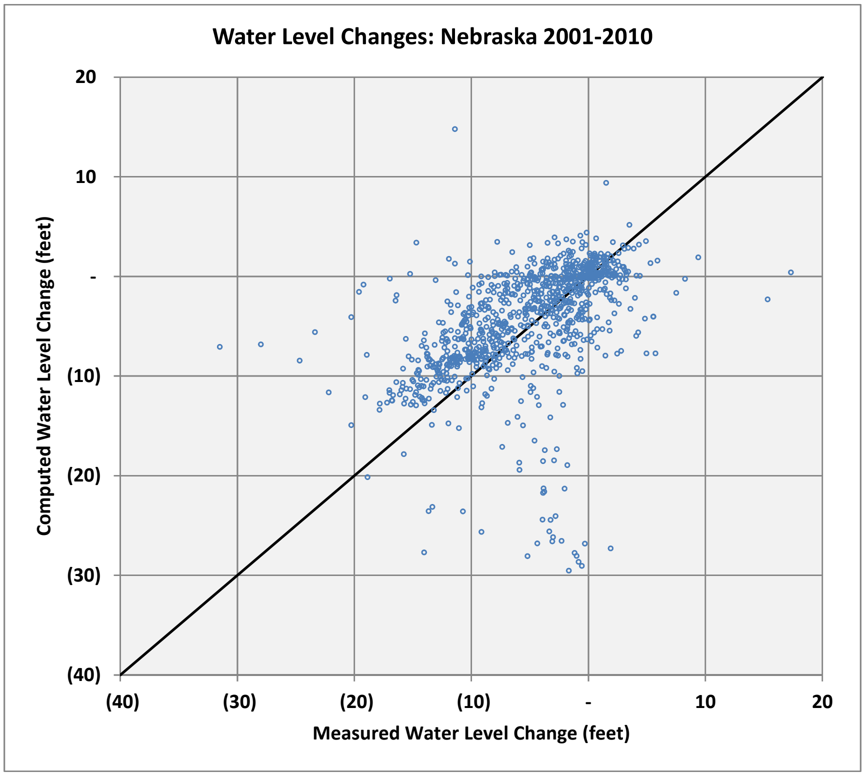


Figure 6a: Scatter Diagram of Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Nebraska.

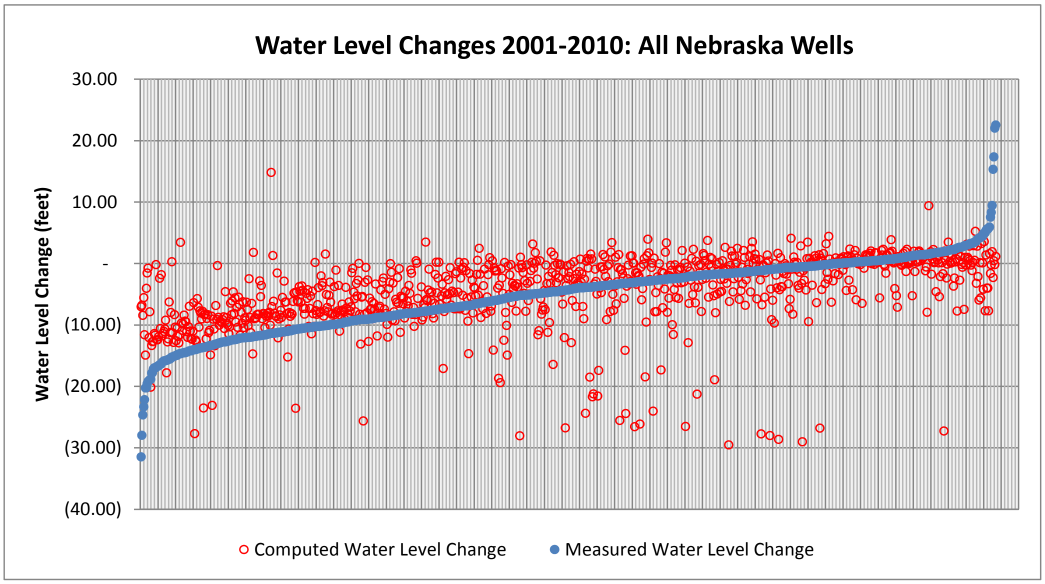


Figure 6b: Well Series Plot Comparing Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Nebraska.

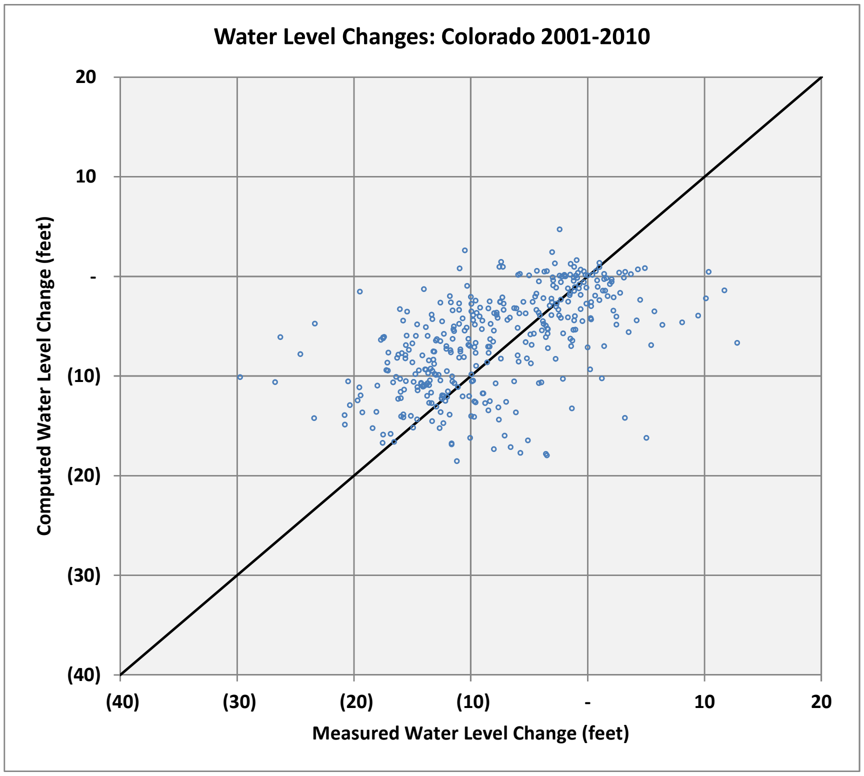


Figure 7a: Scatter Diagram of Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Colorado.

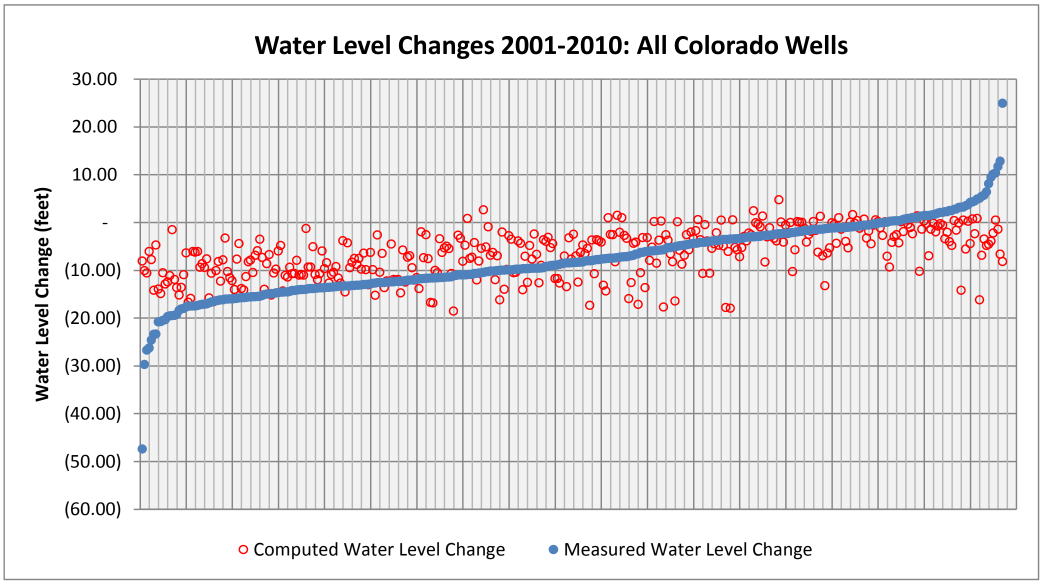


Figure 7b: Well Series Plot Comparing Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Colorado.

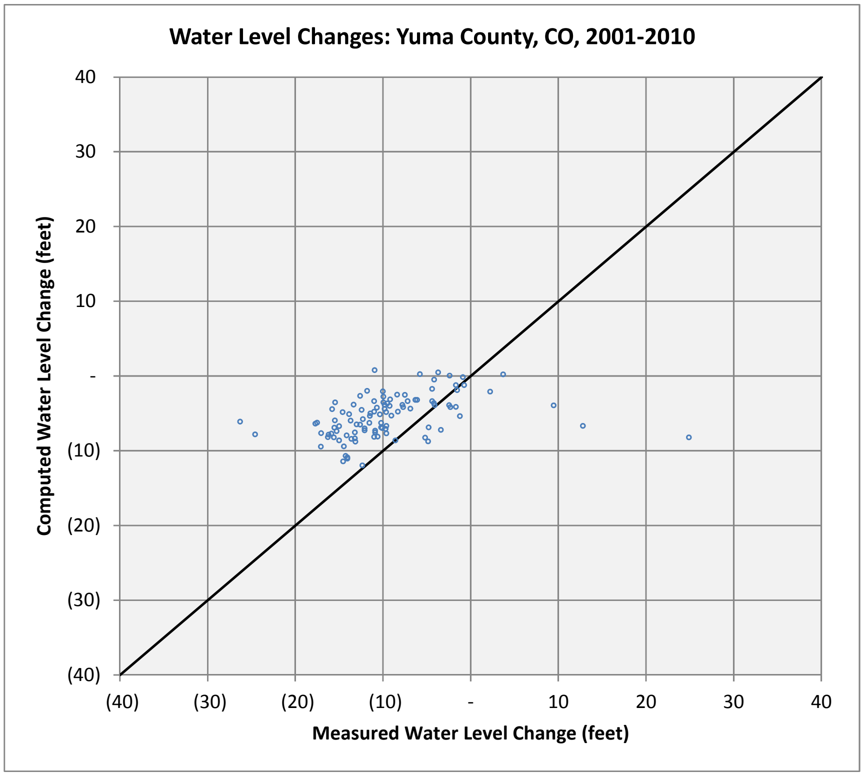


Figure 8a: Scatter Diagram of Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Yuma County, Colorado.

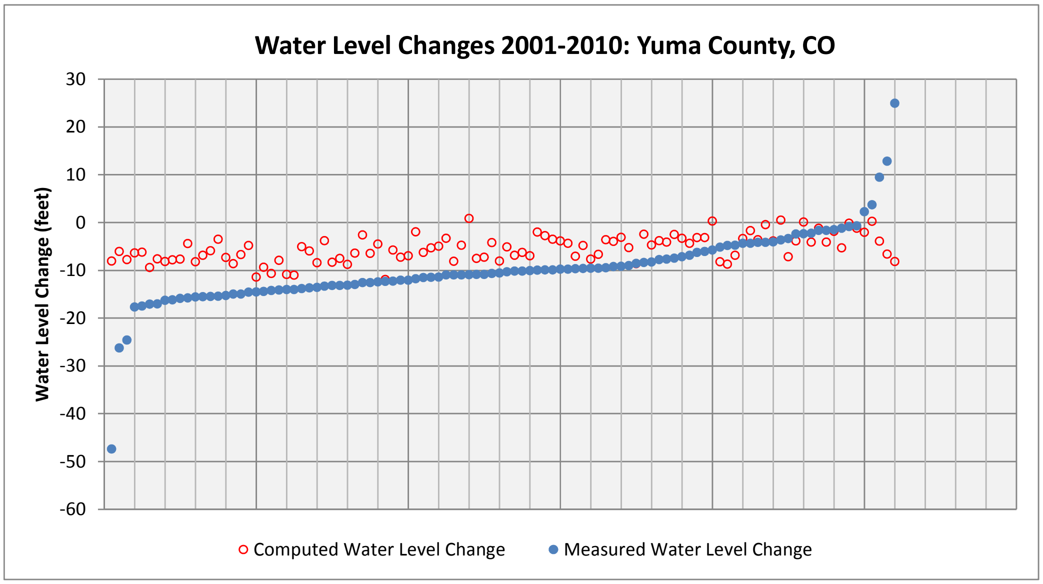


Figure 8b: Well Series Plot Comparing Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Yuma County, Colorado.

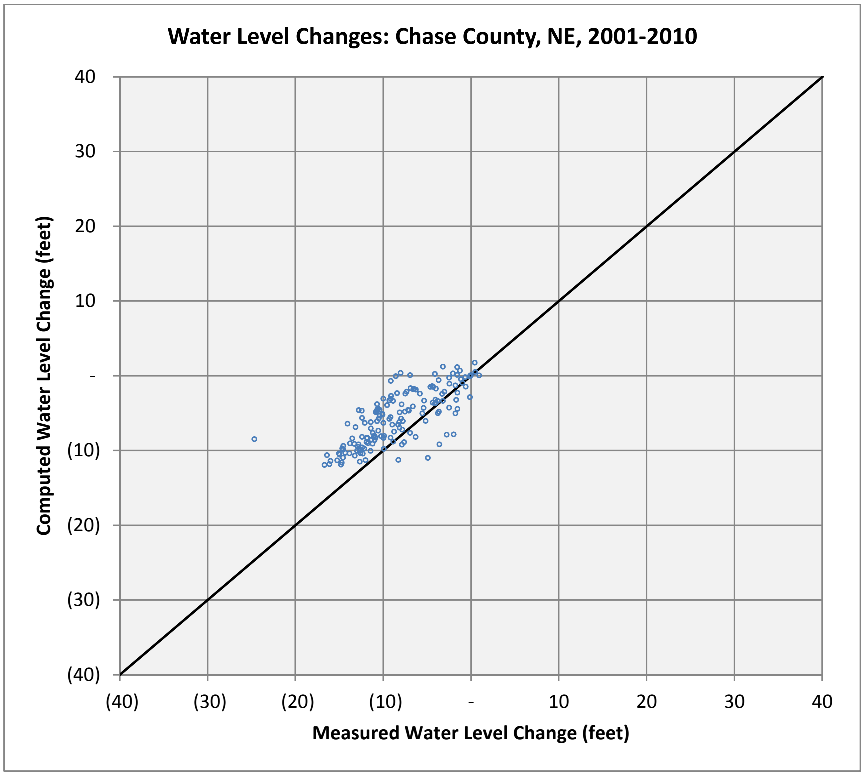


Figure 9a: Scatter Diagram of Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Chase County, Nebraska.

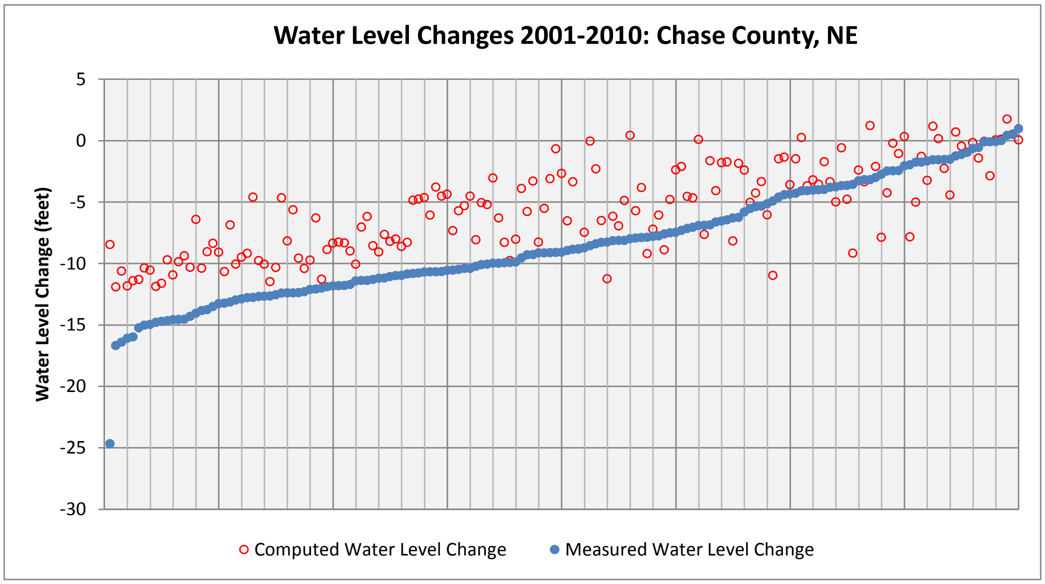


Figure 9b: Well Series Plot Comparing Computed and Measured Groundwater Level Changes from 2001 to 2010 for Wells in Chase County, Nebraska.

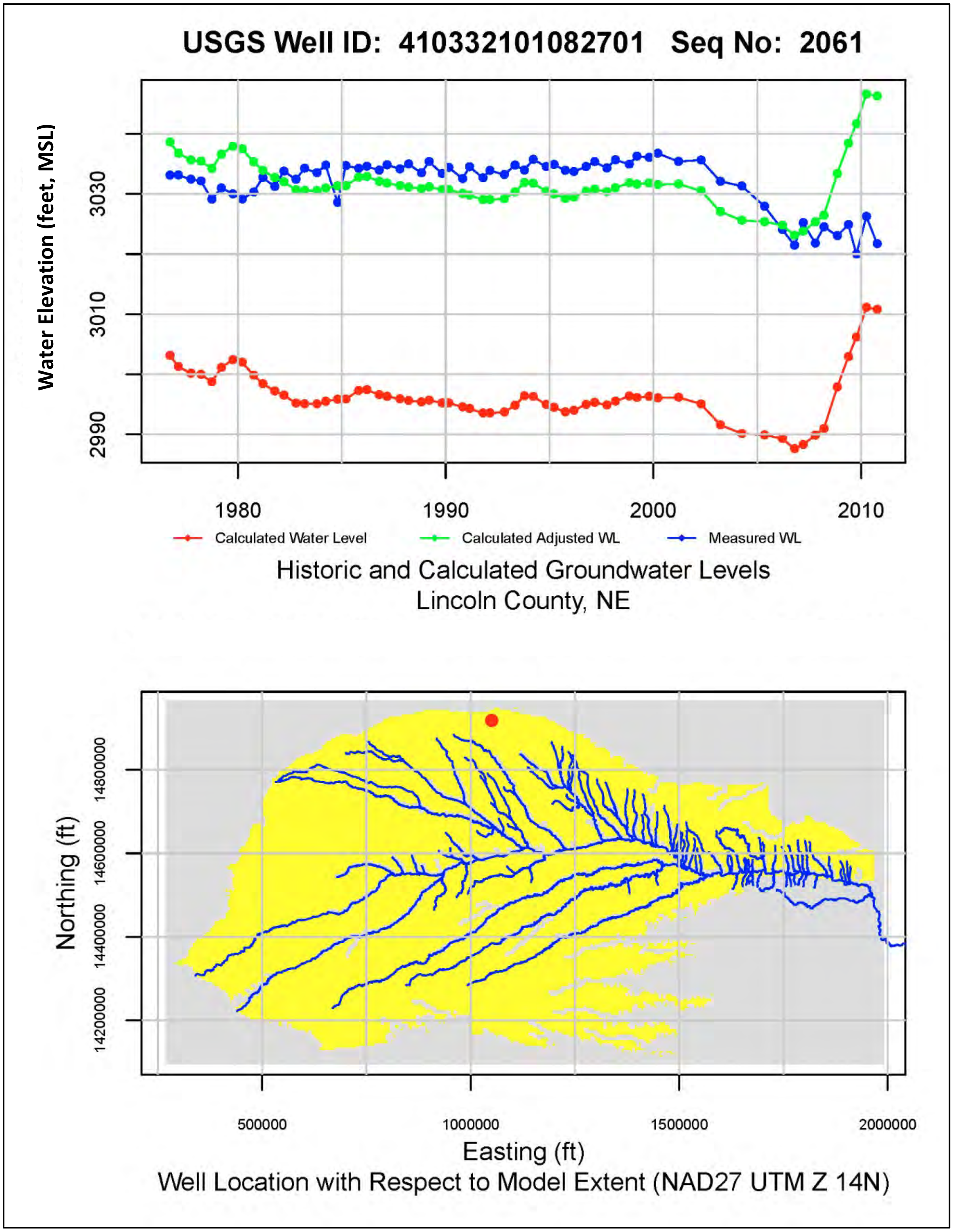


Figure 10: Example Hydrograph Comparing Computed and Measured Groundwater Levels for Well 410332101082701 located in Lincoln County, Nebraska.

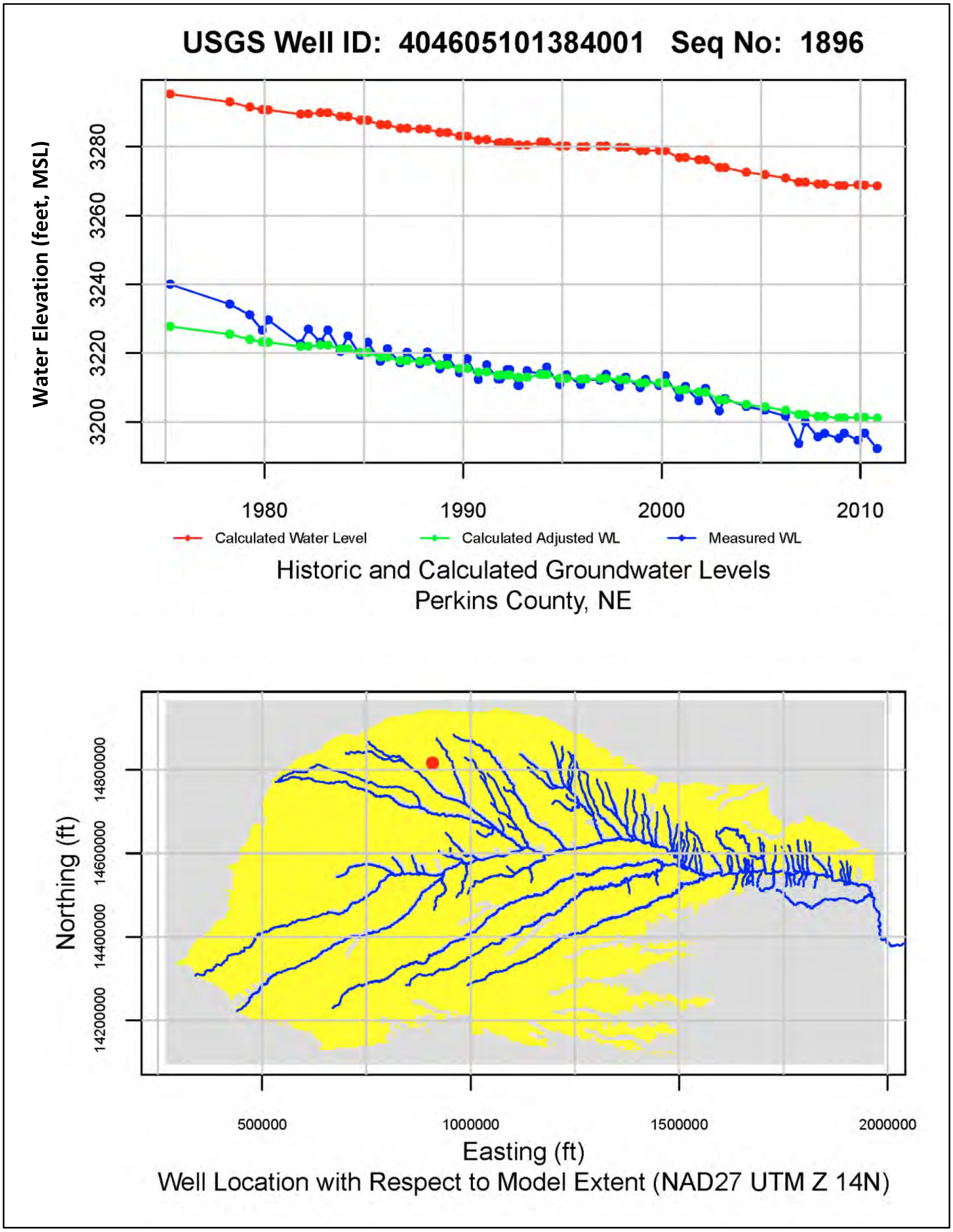


Figure 11: Example Hydrograph Comparing Computed and Measured Groundwater Levels for Well 404605101384001 Located in Perkins County, Nebraska.

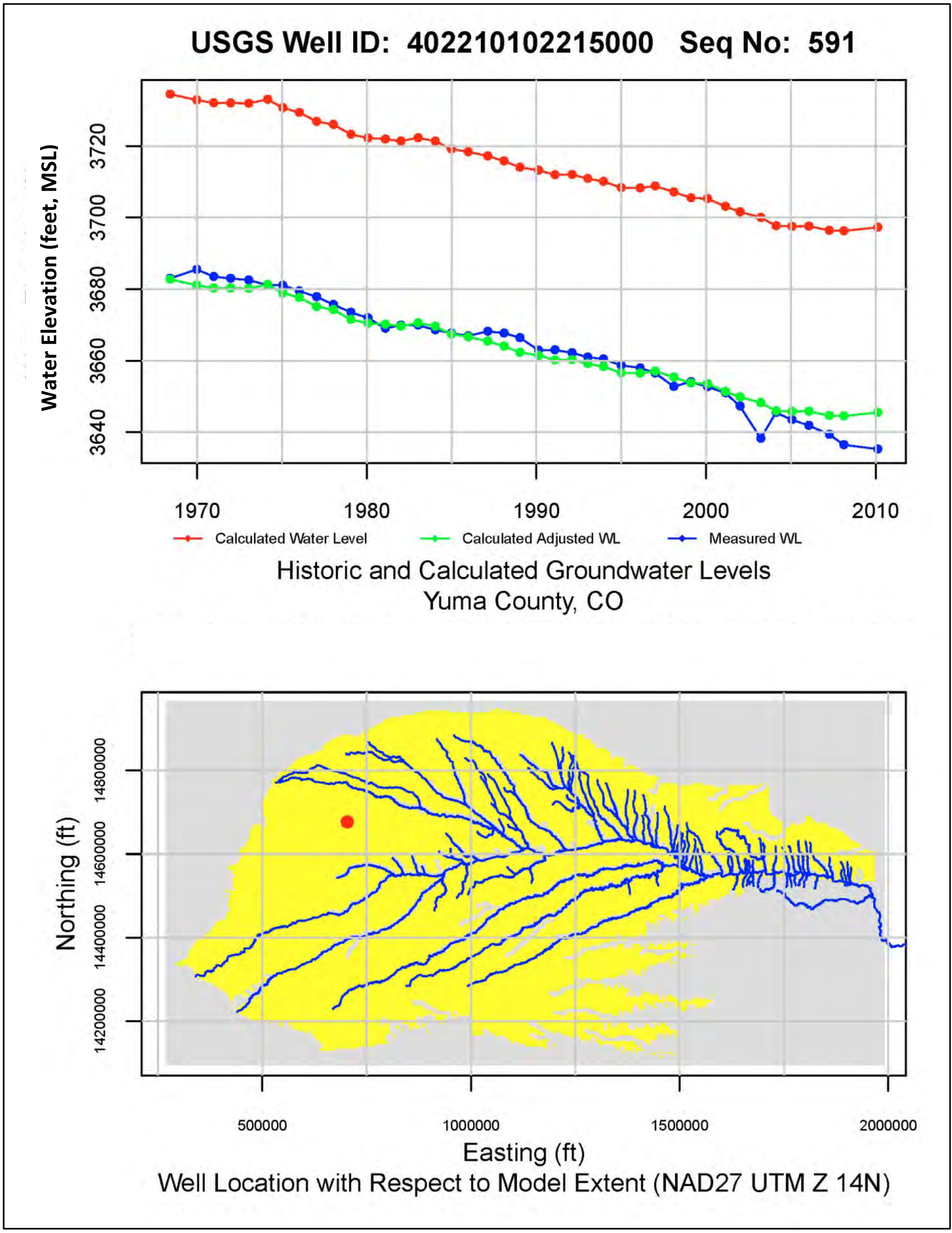


Figure 12: Example Hydrograph Comparing Computed and Measured Groundwater Levels for Well 402210102215000 Located in Yuma County, Colorado.

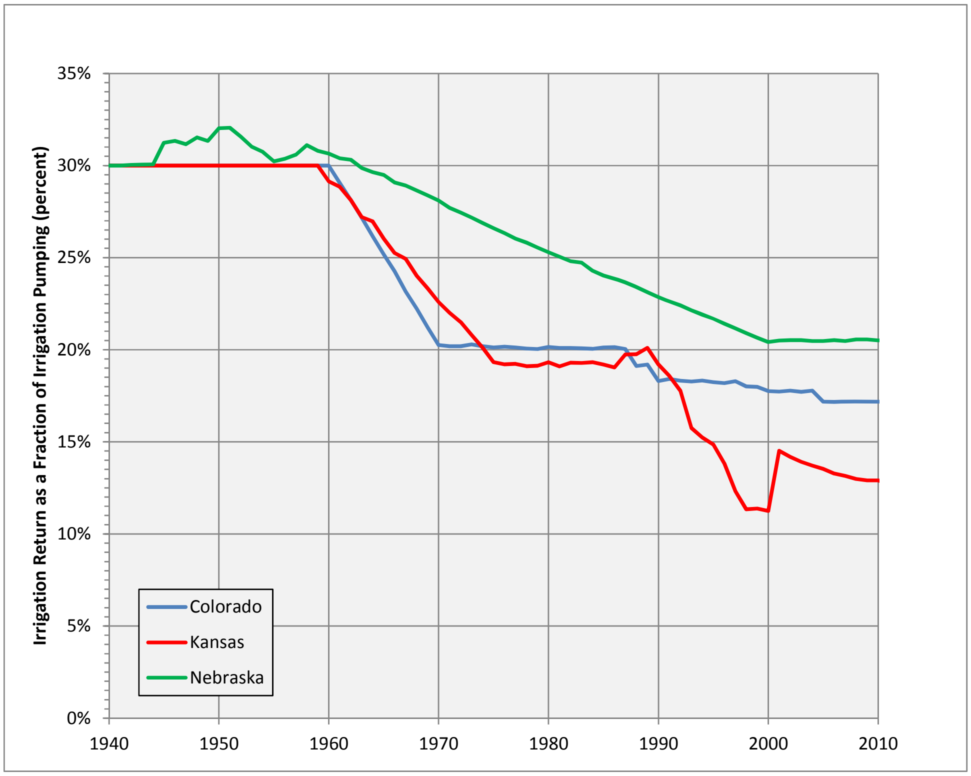


Figure 13: Graph Showing Irrigation Return Flows in Kansas, Nebraska, and Colorado as a Fraction of Irrigation Pumping.

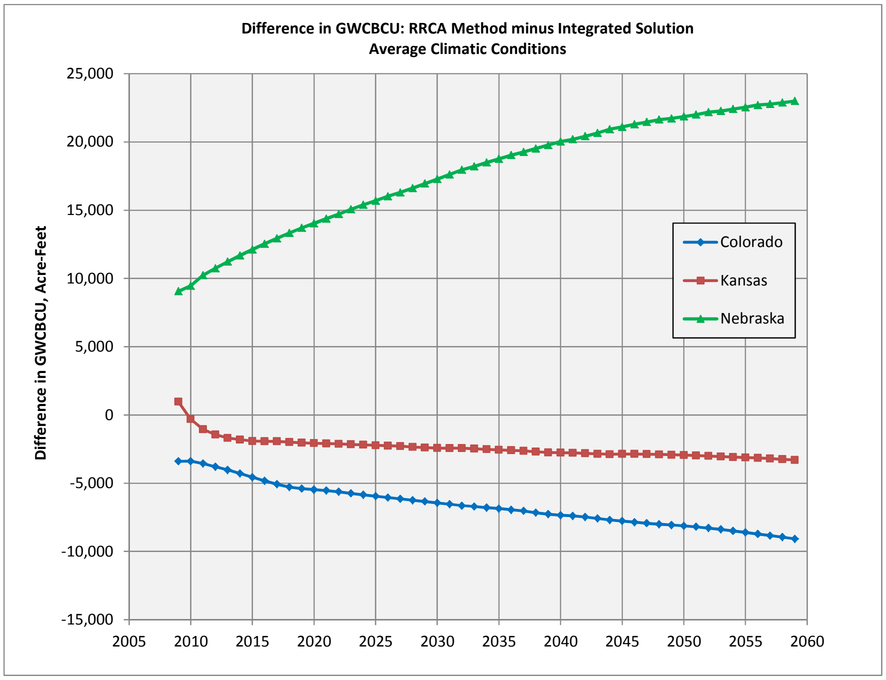


Figure 14a: Difference in GWCBCU – Average Climatic Conditions: RRCA Method minus Integrated Solution.

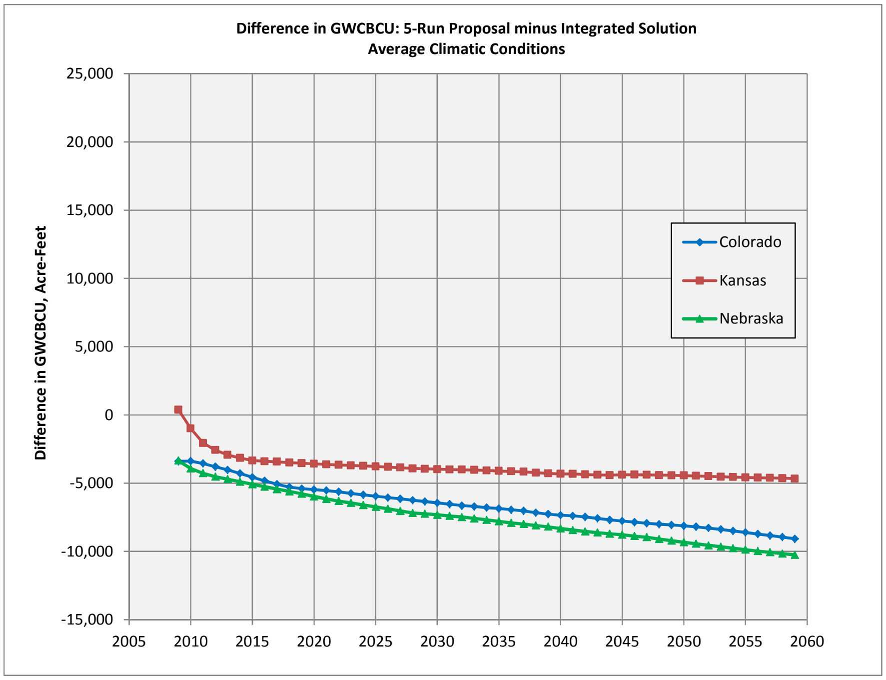
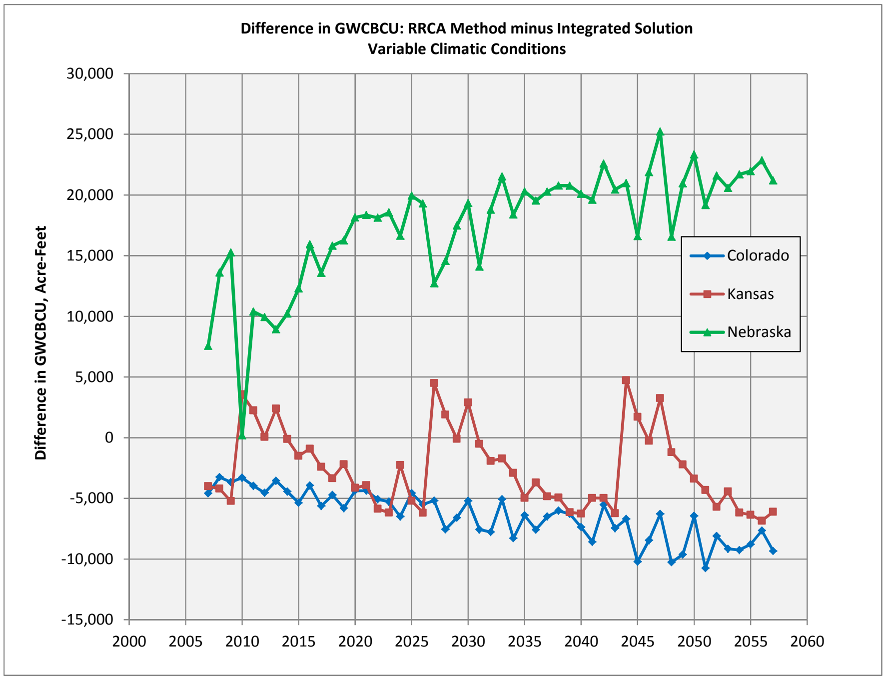


Figure 14b: Difference in GWCBCU – Average Climatic Conditions: 5-Run Proposal minus Integrated Solution.



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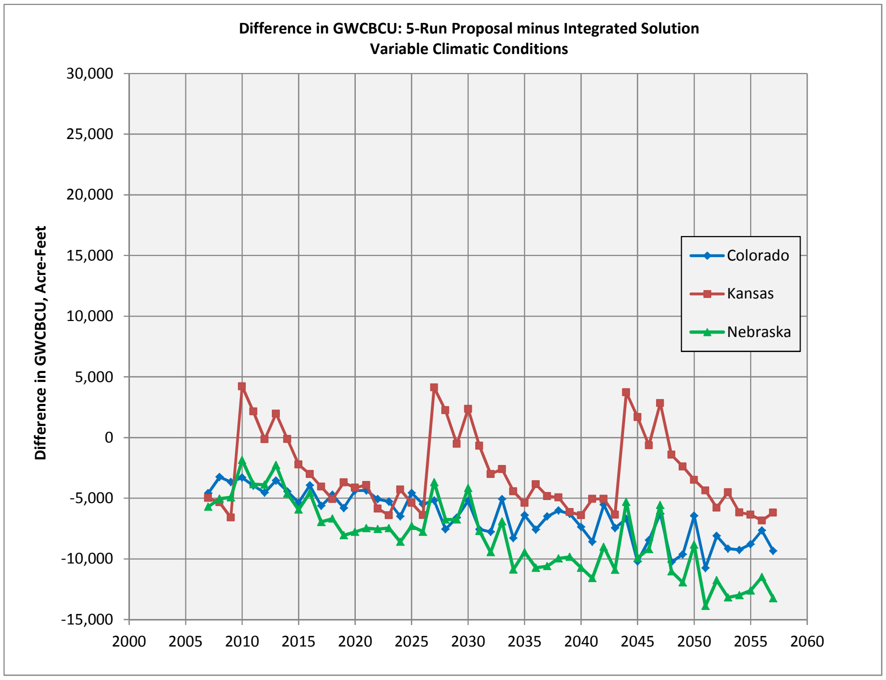

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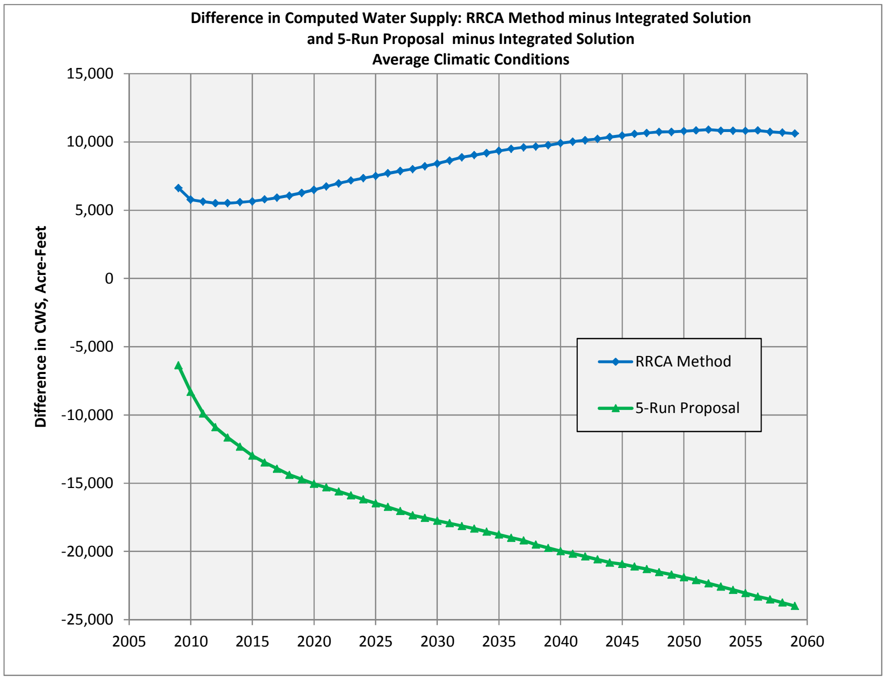

Figure 16: Difference in Computed Water Supply – Average Climatic Conditions:
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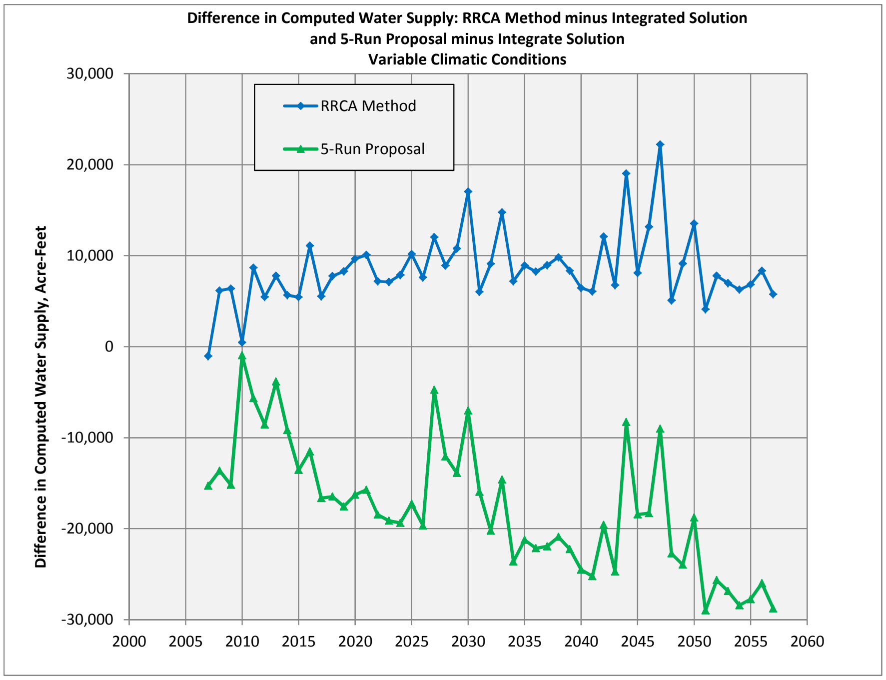

Figure 17: Difference in Computed Water Supply – Variable Climatic Conditions:
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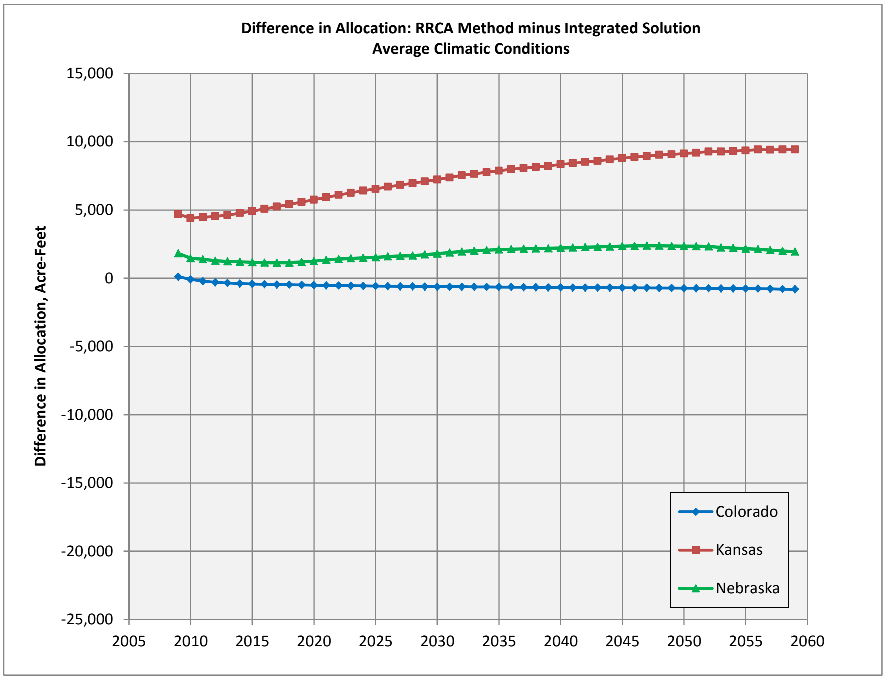

Figure 18a: Difference in Allocation – Average Climatic Conditions:
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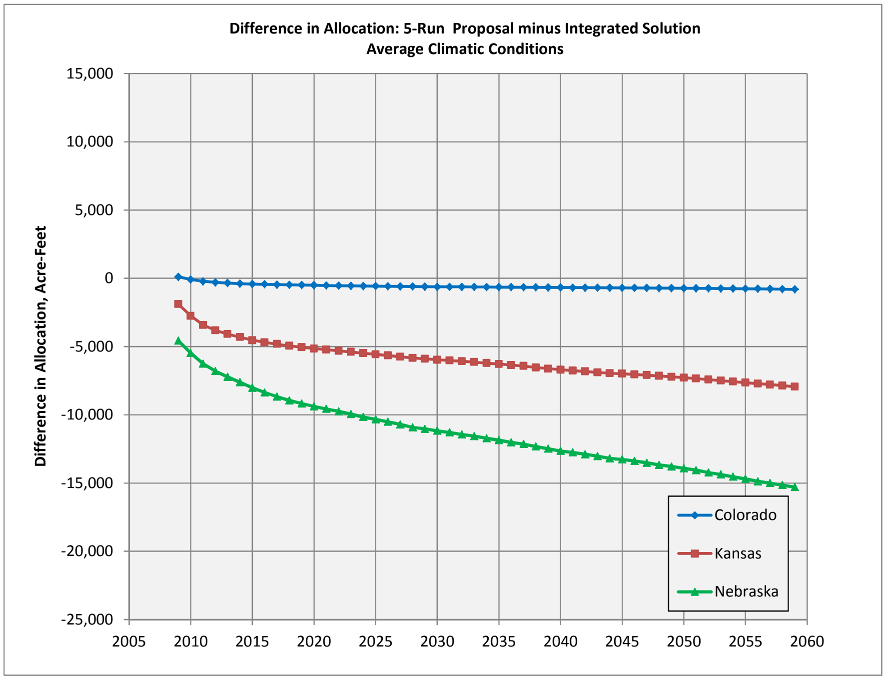

Figure 18b: Difference in Allocation – Average Climatic Conditions:
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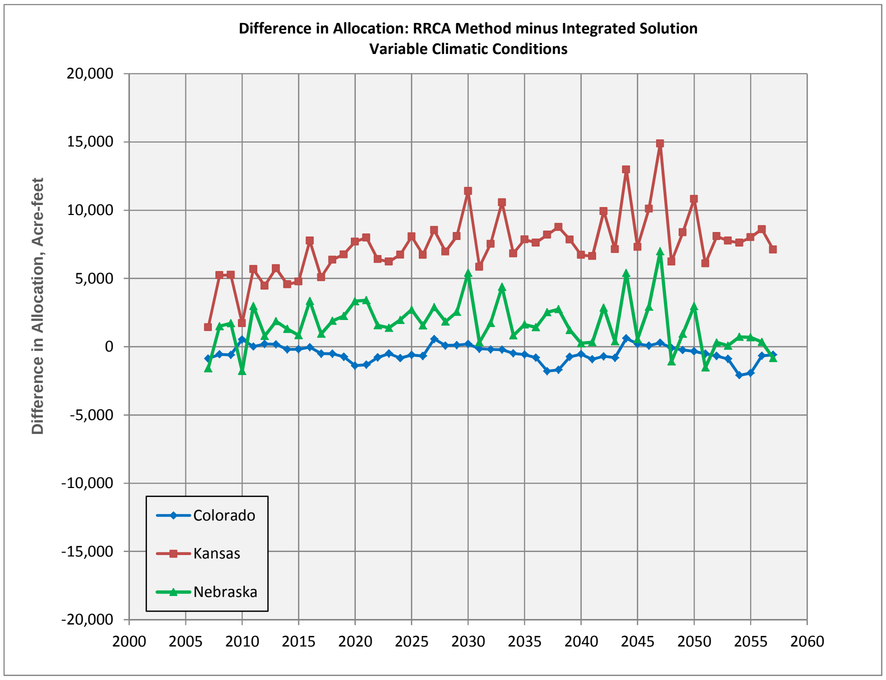

Figure 19a: Difference in Allocation – Variable Climatic Conditions: RRCA Method minus Integrated Solution.

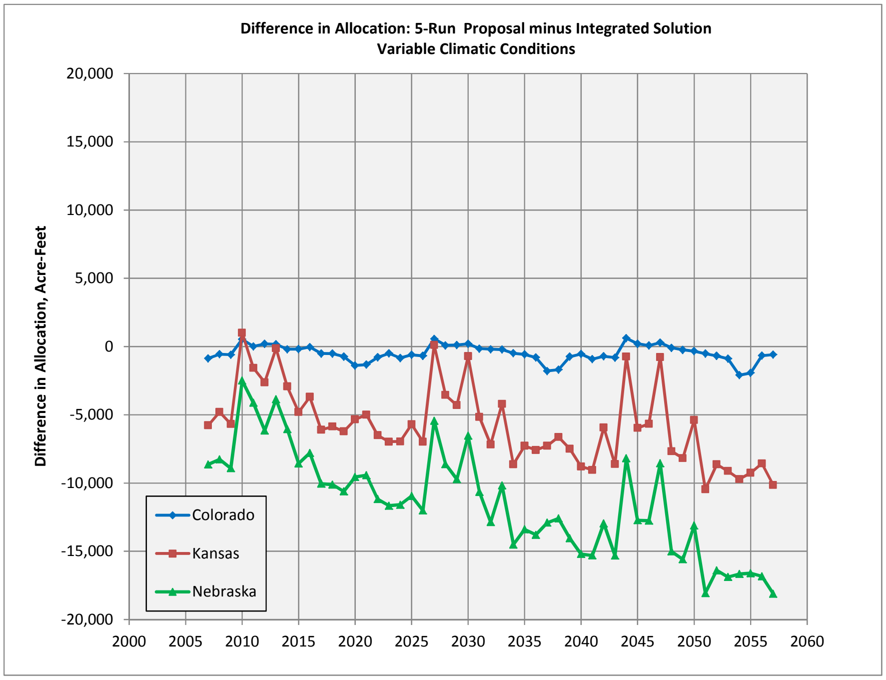

Figure 19b: Difference in Allocation – Variable Climatic Conditions: 5-Run Proposal minus Integrated Solution.

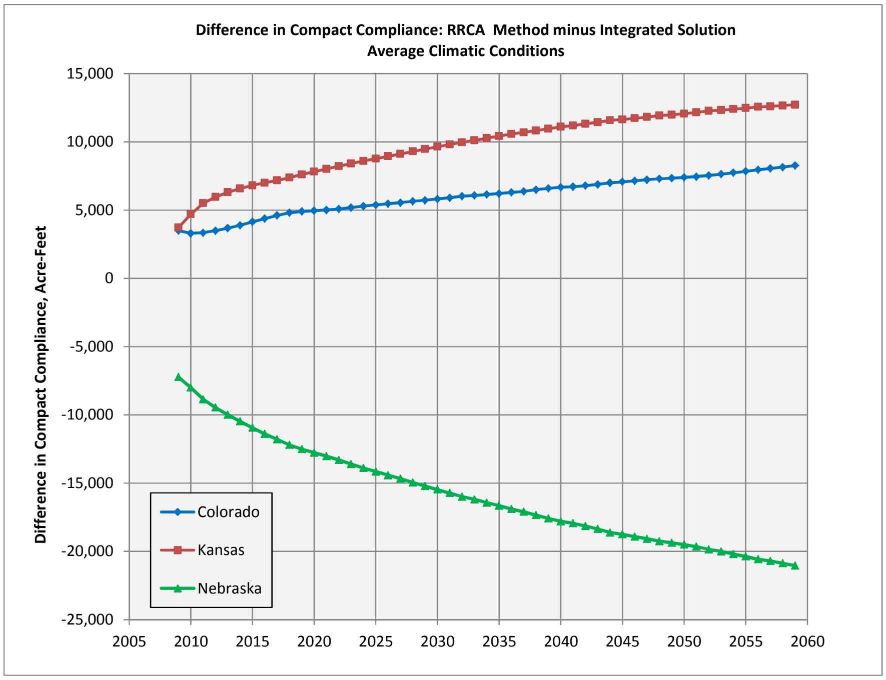

Figure 20a: Difference in Compact Compliance – Average Climatic Conditions: RRCA Method minus Integrated Solution.

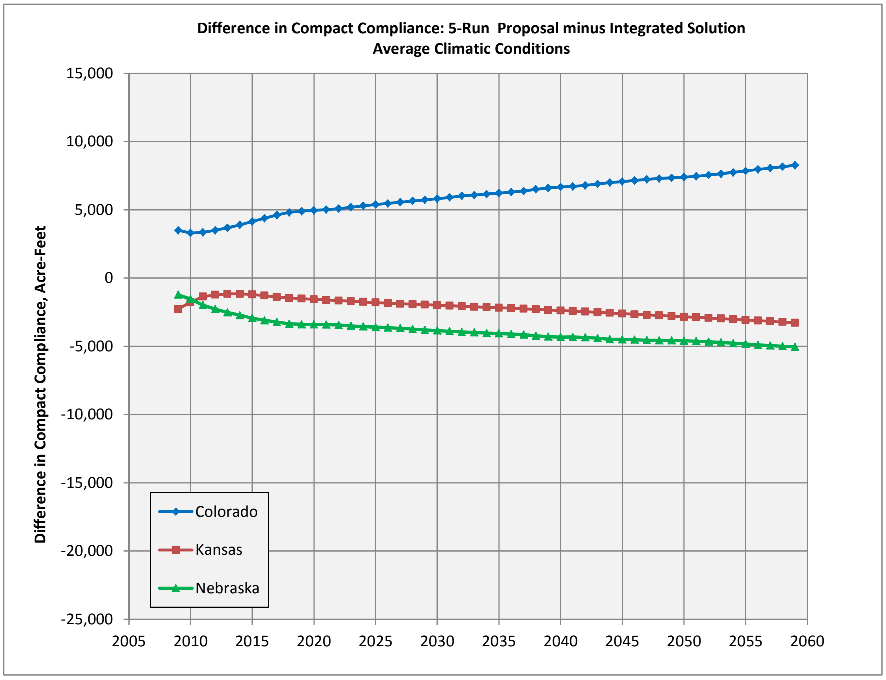

Figure20b: Difference in Compact Compliance – Average Climatic Conditions: 5-Run Proposal minus Integrated Solution.

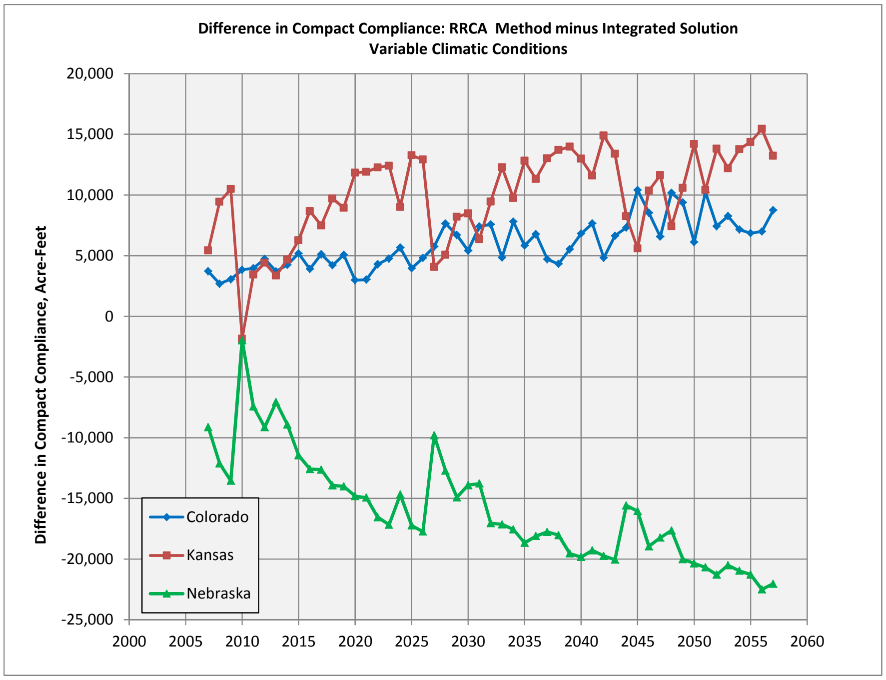

Figure 21a: Difference in Compact Compliance – Variable Climatic Conditions: RRCA Method minus Integrated Solution.

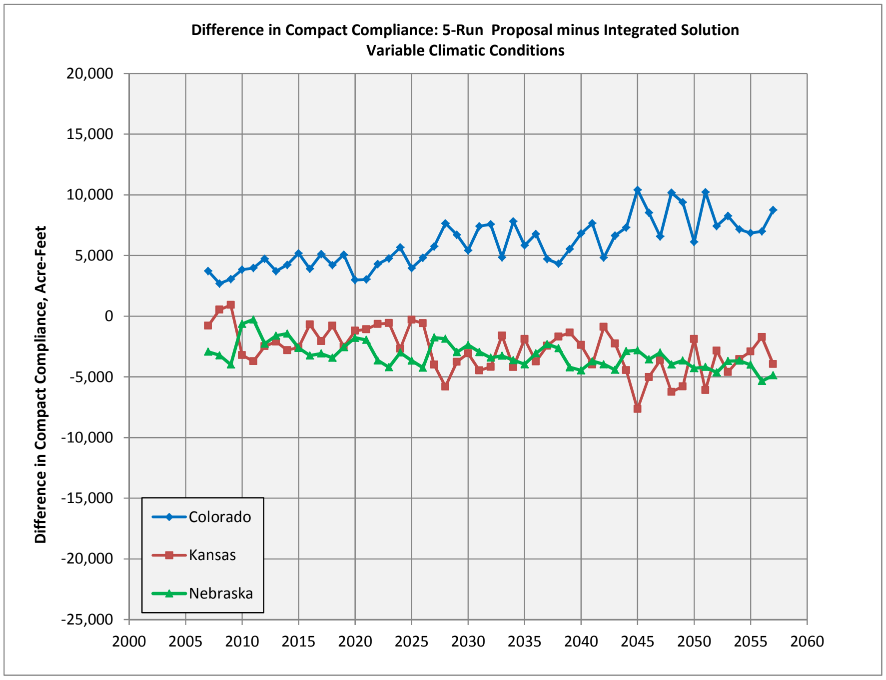

Figure 21b: Difference in Compact Compliance – Variable Climatic Conditions: 5-Run Proposal minus Integrated Solution.

TABLES

Table 1: Compilation of Irrigation Pumping and Irrigation Return Flow from 1940 to 2010 as Reported to the RRCA by Colorado, Kansas, and Nebraska.

Year	Irrigation Pumping			Irrigation Return Flow			Fraction of Irrigation Return Flow		
	Colorado	Kansas	Nebraska	Colorado	Kansas	Nebraska	Colorado	Kansas	Nebraska
1940	1,346	3,405	37,411	404	1,022	11,223	30.0%	30.0%	30.0%
1941	1,402	2,732	30,894	421	820	9,268	30.0%	30.0%	30.0%
1942	1,900	4,099	32,301	570	1,230	9,702	30.0%	30.0%	30.0%
1943	2,819	5,409	43,642	846	1,623	13,112	30.0%	30.0%	30.0%
1944	2,901	4,541	36,628	870	1,362	11,008	30.0%	30.0%	30.1%
1945	2,302	5,150	52,091	691	1,545	16,272	30.0%	30.0%	31.2%
1946	3,616	6,287	54,937	1,085	1,886	17,215	30.0%	30.0%	31.3%
1947	5,655	5,852	44,167	1,697	1,756	13,761	30.0%	30.0%	31.2%
1948	8,355	5,494	34,217	2,507	1,648	10,786	30.0%	30.0%	31.5%
1949	10,818	5,524	41,134	3,245	1,657	12,887	30.0%	30.0%	31.3%
1950	13,987	7,644	20,814	4,196	2,293	6,665	30.0%	30.0%	32.0%
1951	13,382	4,957	16,403	4,015	1,487	5,257	30.0%	30.0%	32.0%
1952	25,658	13,136	37,180	7,697	3,941	11,734	30.0%	30.0%	31.6%
1953	26,343	14,166	46,239	7,903	4,250	14,344	30.0%	30.0%	31.0%
1954	38,708	22,522	79,373	11,612	6,757	24,396	30.0%	30.0%	30.7%
1955	53,198	36,986	92,698	15,959	11,096	28,021	30.0%	30.0%	30.2%
1956	81,808	66,607	149,712	24,542	19,982	45,452	30.0%	30.0%	30.4%
1957	63,839	58,011	167,480	19,152	17,403	51,212	30.0%	30.0%	30.6%
1958	64,301	66,158	124,591	19,290	19,848	38,756	30.0%	30.0%	31.1%
1959	97,655	85,436	275,470	29,297	25,631	84,847	30.0%	30.0%	30.8%
1960	90,282	94,722	268,480	27,085	27,606	82,263	30.0%	29.1%	30.6%
1961	87,671	79,315	222,413	25,472	22,883	67,587	29.1%	28.9%	30.4%
1962	86,757	73,950	135,464	24,392	20,799	41,059	28.1%	28.1%	30.3%
1963	142,954	123,345	314,917	38,842	33,548	94,064	27.2%	27.2%	29.9%
1964	214,073	184,225	331,942	56,005	49,665	98,372	26.2%	27.0%	29.6%
1965	155,393	140,506	277,703	39,119	36,564	81,890	25.2%	26.0%	29.5%
1966	275,095	205,817	378,582	66,726	51,962	110,059	24.3%	25.2%	29.1%
1967	362,137	231,738	443,896	83,885	57,769	128,330	23.2%	24.9%	28.9%
1968	464,363	263,347	604,914	103,235	63,225	173,241	22.2%	24.0%	28.6%
1969	531,688	303,593	584,179	112,737	70,795	165,793	21.2%	23.3%	28.4%
1970	601,750	370,066	903,174	121,851	83,594	253,722	20.2%	22.6%	28.1%
1971	630,744	424,963	881,976	127,341	93,492	244,300	20.2%	22.0%	27.7%
1972	572,578	360,368	811,664	115,603	77,507	222,780	20.2%	21.5%	27.4%
1973	609,592	454,316	888,881	123,701	94,498	241,517	20.3%	20.8%	27.2%
1974	928,838	549,301	1,220,349	187,474	110,465	328,054	20.2%	20.1%	26.9%
1975	880,638	462,786	1,264,274	177,226	89,413	336,215	20.1%	19.3%	26.6%
1976	1,006,365	777,187	1,621,818	202,940	149,289	426,919	20.2%	19.2%	26.3%
1977	919,057	520,707	1,246,573	184,955	100,154	324,442	20.1%	19.2%	26.0%
1978	1,032,721	672,554	1,689,826	207,185	128,514	436,185	20.1%	19.1%	25.8%
1979	834,075	441,661	1,182,022	167,133	84,477	301,894	20.0%	19.1%	25.5%
1980	855,547	534,424	1,549,146	172,352	103,277	391,823	20.1%	19.3%	25.3%
1981	875,096	561,600	1,111,538	175,911	107,236	278,400	20.1%	19.1%	25.0%
1982	662,140	420,593	1,036,258	133,088	81,149	257,066	20.1%	19.3%	24.8%
1983	654,017	466,787	1,204,466	131,304	90,008	297,833	20.1%	19.3%	24.7%
1984	818,038	519,377	1,491,538	163,959	100,370	362,420	20.0%	19.3%	24.3%
1985	684,041	474,299	1,368,050	137,674	91,062	328,743	20.1%	19.2%	24.0%
1986	721,067	552,279	1,390,985	145,232	105,144	331,735	20.1%	19.0%	23.8%
1987	756,271	431,503	1,301,147	151,531	85,169	307,768	20.0%	19.7%	23.7%
1988	847,765	464,451	1,639,301	162,054	91,737	383,603	19.1%	19.8%	23.4%
1989	711,202	532,617	1,514,249	136,469	107,069	349,833	19.2%	20.1%	23.1%
1990	743,432	512,588	1,718,934	136,075	98,395	392,986	18.3%	19.2%	22.9%
1991	670,431	477,883	1,908,252	123,489	88,768	431,798	18.4%	18.6%	22.6%
1992	696,201	263,613	1,123,510	127,509	46,869	251,702	18.3%	17.8%	22.4%
1993	654,381	255,110	549,078	119,628	40,179	121,570	18.3%	15.7%	22.1%
1994	827,192	392,065	1,519,332	151,617	59,726	332,647	18.3%	15.2%	21.9%
1995	680,446	366,239	1,752,046	124,131	54,428	380,042	18.2%	14.9%	21.7%
1996	594,535	364,518	1,101,024	108,090	50,396	235,820	18.2%	13.8%	21.4%
1997	721,848	414,693	1,758,118	131,998	51,085	372,074	18.3%	12.3%	21.2%
1998	744,589	382,800	1,604,741	134,098	43,380	335,454	18.0%	11.3%	20.9%
1999	643,548	333,959	1,178,570	115,724	38,007	243,417	18.0%	11.4%	20.7%
2000	901,788	495,708	2,245,099	160,118	55,777	458,496	17.8%	11.3%	20.4%
2001	876,397	451,543	1,774,258	155,370	65,559	363,542	17.7%	14.5%	20.5%
2002	906,631	569,053	2,495,095	161,160	80,733	511,836	17.8%	14.2%	20.5%
2003	890,479	520,436	2,149,726	157,780	72,387	441,155	17.7%	13.9%	20.5%
2004	730,747	518,221	1,891,480	129,883	71,050	387,084	17.8%	13.7%	20.5%
2005	724,983	425,789	1,660,810	124,574	57,620	339,850	17.2%	13.5%	20.5%
2006	761,664	464,920	1,673,444	130,791	61,700	343,381	17.2%	13.3%	20.5%
2007	650,306	446,107	1,367,300	111,738	58,658	279,886	17.2%	13.1%	20.5%
2008	660,933	432,559	1,418,878	113,613	56,153	291,758	17.2%	13.0%	20.6%
2009	522,724	324,167	1,288,940	89,798	41,839	264,932	17.2%	12.9%	20.6%
2010	669,739	394,294	1,179,792	115,027	50,884	241,954	17.2%	12.9%	20.5%

Table 2: Comparison of the Fractions of Irrigation Return Flow from 2001 to 2010 as Reported to the RRCA by Colorado, Kansas, and Nebraska.

Year	Fraction of Irrigation Return Flow			Difference of Irrigation Return Flow Fraction		Corresponding Amount of Return Flow (acre-feet)	
	Colorado	Kansas	Nebraska	Colorado minus Kansas	Nebraska minus Kansas	Colorado	Nebraska
2001	17.7%	14.5%	20.5%	3.2%	6.0%	28,127	105,941
2002	17.8%	14.2%	20.5%	3.6%	6.3%	32,534	157,850
2003	17.7%	13.9%	20.5%	3.8%	6.6%	33,923	142,151
2004	17.8%	13.7%	20.5%	4.1%	6.8%	29,694	127,755
2005	17.2%	13.5%	20.5%	3.7%	6.9%	26,466	115,101
2006	17.2%	13.3%	20.5%	3.9%	7.2%	29,711	121,298
2007	17.2%	13.1%	20.5%	4.0%	7.3%	26,230	100,103
2008	17.2%	13.0%	20.6%	4.2%	7.6%	27,812	107,564
2009	17.2%	12.9%	20.6%	4.3%	7.6%	22,333	98,575
2010	17.2%	12.9%	20.5%	4.3%	7.6%	28,597	89,702

Appendix B

Appendix B

Detailed Description of the Integrated Solution

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Kansas Department of Agriculture, Division of Water Resources.
May 15, 2013

Table of Contents

Introduction

Appendix B presents an approximate solution for the states' impacts on computed baseflow that sum to the total impact within a tolerance that can be made as small as desired, and appropriately distributes pumping impacts among the states. We refer to this approximation as the integrated solution, which can be specified to either include or exclude imported water from the base case.

The integrated solution addresses the nonlinearity of stream depletion that occurs over the range of pumping due to stream drying; that is, as pumping increases, less streamflow is available to be depleted. The depletion response can be conceptualized as a normally decreasing function over the range of pumping from zero to the total reported pumping by the three States during any given time period. If the range of pumping is subdivided into a large number of slices, or intervals, then for each interval of total pumping we find, as the width of each interval approaches zero, the sum of the States' depletions due to varying each State's pumping separately over the range of the interval equals the total impact due to varying all of the States' pumping together. That is, as the intervals get smaller, the sum of the States' impacts becomes a better approximation for the total impact over that interval, and an exact solution for the States' impacts exists at the limit as the width of the intervals approaches zero. At that limit, the sum over the incremental impacts is represented as a continuous integration to give the exact solution.

Two techniques are used to give a good approximation for the exact solution. The mathematical basis for these techniques is described below, under Methods. One technique is to subdivide the range of pumping into intervals, and to calculate impacts over the range of each interval. A second technique is to represent the average depletion response over the range of each interval in calculating the impacts. To illustrate this technique, consider the full range of pumping as a single interval. The average depletion response for each State's impact can be applied to this interval by taking the average of two cases: first, the impact of turning the State's pumping off with all other pumping on; and second, the impact of turning the State's pumping on with all other pumping off.

Similarly, this averaging technique can be applied for any number of intervals. With ten stress intervals, each State's impact is calculated over ten-percent intervals of pumping in two "directions" and then averaged. For the interval between 90 and 100 percent of total pumping, each State's impact is calculated (a) with the State's pumping at 90 percent, holding the other States' pumping at 100 percent; and (b) with the State's pumping at 100 percent, holding the other States' pumping at 90 percent. The average depletion response over this interval is applied by taking the average of impacts according to stress decrements as in (a) and stress increments as in (b).

The integrated solution applies these techniques to approximate the exact solution as closely as desired. The solution is described in the next section and then demonstrated with an example that applies ten-percent stress intervals and average response functions to historical conditions for years 1918-2010. Results for this example show that the discrepancy with respect to the total impact is negligible.

Appendix B ends by presenting stream depletion characteristics over the full range of pumping for the total basin and for several accounting subbasins. These characteristics are a result of the integrated solution applied to the scenario of average future conditions described in the Expert Report and projected to year 2059.

Method

The mathematical basis of the integrated solution is the total differential, a concept from calculus first introduced by Gottfried Leibniz in 1684 (see Zeidler et al., 2004; Oboukhoff, 1940). The total differential states in effect that, at any point on a function, the total incremental change in the function equals the sum of impacts due to small variations in its independent variables in the limit as the magnitude of those variations approaches zero. This concept is commonly applied in uncertainty analysis (Wikipedia, 2012).

The total differential of a function can be visualized as a tangent that equals the function at a single point on the function. The tangent is a line for a function of a single independent variable, and a plane for a function of two variables; the concept generalizes to more than two variables but is difficult to visualize. With imported water supply (IWS) excluded from the base case, computed baseflow is treated as a function of a single variable to calculate the total impact of pumping, and as function of three variables to calculate the impacts of each state's pumping. Alternatively, computed baseflow can be treated as a function of four variables if IWS is included in the base case.

The total differential is approximated to calculate the total impact of pumping on computed baseflow for small but discrete variations in pumping by the three states. This is accomplished with the RRCA groundwater model with no model changes over the full range of pumping. The discretized total differential is then integrated by summing the impacts over the pumping intervals.

For computed baseflow with no IWS in the base case, the terms of the total differential, referred to as partial differentials, are represented by the product of a partial derivative and a change in the independent variable. The partial derivatives quantify how stream depletion varies over the range of pumping by each state. It is this variation that the integrated solution addresses in order to eliminate the approximation error from the calculation of the states' impacts.

The integrated solution can improve accuracy through two approaches. First, the width of stress intervals can be reduced, with corresponding increases in the number of pumping intervals and model runs. Second, the impacts calculated for each pumping interval can be approximated using either forward or central differences, as in the next section. Forward differences can be applied using either stress increments or decrements, and approximation error is roughly a linear function of stress interval. Impacts of both stress increments and decrements are calculated using a version of Willem's acct program.

Central differences greatly improve the approximation of impacts compared with forward differences, and can be expressed as the average of forward difference approximations based on stress increments and decrements. The central difference approximation is applied as a spreadsheet operation to average impacts calculated for stress increments and decrements. Using these techniques, the integrated solution is both computationally practical and accurate.

Parameterization of stress intervals

The stress fraction, *f*, is defined to parameterize stress intervals $h_i = fP_i$ for the *i*th stress, and $h = fP$ for the jointly applied stresses, so that as the combined or individual stress fraction varies between 0 and 1, the combined stress *P* or individual stress P_i varies from no pumping to full pumping. The number of equal discretization intervals $n = 1/f$; and for a given discretization, the *k*th stress fraction $f_k = k/n$ for *k* from 0 to *n*. This parameterization is used to discretize the integration of the total differential over the range of pumping as a summation of impacts over stress fractions from 0 to 1.

Model runs corresponding to a given set of stress fractions are specified using the command STRESSF, which was added to a version of the Republican River Preprocessor named rrpptestv4. Specified stress fractions can be passed to the program as command line arguments. This feature is used in batch procedures to automate the required model runs for a sequence of stress intervals.

Discretization of total differential

The total differential of computed baseflow as a function of *m* independent variables is approximated by

$$\Delta Q_k \approx \frac{\partial Q_k}{\partial P_1}\Delta P_1 + \frac{\partial Q_k}{\partial P_2}\Delta P_2 + \ldots + \frac{\partial Q_k}{\partial P_m}\Delta P_m, \text{or } \Delta Q_k = \sum_{i=1}^{m}\frac{\partial Q_k}{\partial P_i}\Delta P_i \; , \tag{1}$$

where ΔP_i represents a small but finite change in each stress. With IWS excluded from the base case, the number of independent variables *m*=3, and only pumping stresses are considered in Equation (1).

The subscript *k* indicates that each term in Equation (1) represents the impact of varying the associated stress with respect to baseflow conditions in which pumping is held at a fraction f_k of full pumping, where $f_k = k/n$. For example, if pumping is discretized into ten steps (*n*=10) , then for k=9, the impact of variation in each state's pumping is calculated with respect to a base case that includes 90 percent of all pumping in the model.

Each partial derivative, $\partial Q_k / \partial P_i$ in Equation (1) represents the depletion response to change in a given stress, P_i, holding the other stresses constant at conditions defined for the kth stress fraction. As the increments ΔP_i approach zero, the sum of terms given by Equation (1) converges to the total differential. At this limit Equation (1) is linear, i.e. its terms are additive, and the sum of terms equals the impact of the pumping stresses combined into a single variable. With IWS included in the base case, computed baseflow is treated as a function of four independent variables corresponding to the stresses of the three states' pumping and the imported water supply. If IWS is excluded from the base case, computed baseflow is treated as a function of three independent variables, $Q(P_1, P_2, P_3)$,where P_1, P_2, P_3 correspond to Colorado, Kansas and Nebraska pumping.

Total impact and integration over range of stresses

The total impact for the kth stress interval is calculated independently for comparison with the sum of terms given by Equation (1). For this purpose, computed baseflow is treated as a function of a single variable, $Q(P)$, where P is the sum of the three states' pumping, $P = \sum P_i$. (If IWS had been included in the base case, P would include IWS as a term in the sum of four stresses.) The total depletion response to pumping is represented by the derivative dQ/dP , which is approximated by

$$\frac{\Delta Q}{\Delta P} = \frac{Q(P+\Delta P)-Q(P)}{\Delta P} \tag{2}$$

This is a forward differences approximation that is used in calculus to define the derivative in the limit as ΔP approaches zero. For each stress interval k from 1 to n, the total impact is given by

$$\Delta Q_k = \frac{dQ_k}{dP}\Delta P = Q_k(P+\Delta P) - Q_k(P) \tag{3}$$

The two terms on the right-hand side represent two model runs that differ in total pumping by ΔP , which could be either a stress increment or decrement. The total impact is integrated over the range of pumping by summing the incremental impacts,

$$\Delta Q = \sum_{k=1}^{n} \Delta Q_k \tag{4}$$

The total impact given by Equation (4) does not vary with respect to the number of discretization intervals, n, defined above. This is explained by substituting the right-hand side of Equation (3) into Equation (4), in which case all intermediate terms cancel and only the outside terms corresponding to full pumping and no pumping remain. However, calculating the total impact for each stress interval is useful for comparison with the sum of state pumping impacts given by (1). The total impact over the full range of pumping given by (4) is compared with the integration of (1) over the full range of pumping, which is described next.

Integrating the total differential

The discretized total differential given by (1) is integrated over the full range of stresses between no pumping and total pumping by summing each partial differential in (1) over *k* stress intervals from 1 to n. This integration is denoted by the approximation

$$\Delta Q \approx \sum_{k=1}^{n} \frac{\partial Q_k}{\partial P_1} \Delta P_1 + \sum_{k=1}^{n} \frac{\partial Q_k}{\partial P_2} \Delta P_2 + \ldots + \sum_{k=1}^{n} \frac{\partial Q_k}{\partial P_m} \Delta P_m \ . \tag{5}$$

The states' pumping impacts for the *k*th stress interval in each summation on the right-hand side of Equation (5) are calculated with respect to base case conditions defined for the *k*th stress fraction. The discrepancy associated with the integrated solution is given by the difference between the sum of terms in Equation (5) and the total impact according to Equation (4).

Finite difference approximations

The approximation error in Equation (1) depends both on the discretization interval, ΔP_i, and on how the partial derivatives are approximated. The principal ways to approximate the partial derivatives are by forward and central differences; see, for example, Conte and de Boor (1980).

Forward differences

A forward difference approximation of the partial derivatives in (1) is given by

$$\frac{\partial Q}{\partial P_i} \approx \frac{Q(P_i + \Delta P_i) - Q(P_i)}{\Delta P_i} \tag{6}$$

Then each term in (1) is given by

$$\Delta Q_{i,k} \equiv \frac{\partial Q}{\partial P_i} \Delta P_i \approx Q(P_i + \Delta P_i) - Q(P_i) \tag{7}$$

This approximation is first-order accurate; i.e. the approximation error varies linearly with ΔP_i. Each term of Equation (1) is an incremental impact that is given by (7), and which is evaluated by calculating the differences in computed baseflow between two model runs.

Note that Equation (7) approximates the impact of a stress *increment*, as opposed to the impact of a stress *decrement*, which is expressed by

$$\Delta Q_{i,k} \equiv \frac{\partial Q}{\partial P_i}\Delta P_i \approx Q(P_i - \Delta P_i) - Q(P_i) \qquad (8)$$

Central differences

A central difference approximation of the partial derivatives in (1) is given by

$$\frac{\partial Q}{\partial P_i} \approx \frac{Q(P_i + \Delta P_i) - Q(P_i - \Delta P_i)}{2\Delta P_i} \qquad (9)$$

In this case, each term in (1) is approximated by

$$\frac{\partial Q}{\partial P_i}\Delta P_i \approx \frac{Q(P_i + \Delta P_i) - Q(P_i - \Delta P_i)}{2} \qquad (10)$$

The central difference approximation can often improve an approximation substantially, and turns out to be very useful for our purposes.

The central difference approximation can be implemented as the average of impacts given by two forward difference approximations, where one is calculated from stress decrements and the other from stress increments. Adding and subtracting the term $Q(P_i)$ to the numerator in (6) gives

$$\frac{\partial Q}{\partial P_i}\Delta P_i \approx \frac{\left[Q(P_i + \Delta P_i) - Q(P_i)\right] + \left[Q(P_i) - Q(P_i - \Delta P_i)\right]}{2} \qquad (11)$$

Equation (11) provides a convenient way to apply central differences, which is implemented as the average of impacts taken with respect to either side of a specified interval. Note that the sum over n stress intervals can be evaluated either as a sum over the averages given by (11),

$$\Delta Q_i \approx \sum_{k=1}^{n} \left\{\left[Q(P_{i,k} + \Delta P_i) - Q(P_{i,k})\right] + \left[Q(P_{i,k}) - Q(P_i - \Delta P_{i,k})\right]\right\}/2 \qquad (12)$$

or as an average of sums given by

$$\Delta Q_i \approx (1/2)\left\{\sum_{k=1}^{n}\left[Q(P_{i,k} + \Delta P_i) - Q(P_{i,k})\right] + \sum_{k=1}^{n}\left[Q(P_{i,k}) - Q(P_i - \Delta P_{i,k})\right]\right\} \qquad (13)$$

Equation (13) expresses the central difference approximation as an arithmetic average of two forward difference approximations that correspond to stress increments according to (7) and stress decrements according to (8).

Equation (13) is implemented as follows. First, after making the necessary model runs, the accounting program acct_base_incr, a version of Willem's acct program, is used to evaluate the forward difference approximations separately, once for stress increments and once for stress decrements. Second, the output from the accounting program is imported into Excel, where their arithmetic average is calculated to apply the central difference approximation according to Equation (13).

A version of the accounting program to calculate impacts, acct_basemon_incr, calculates impacts for each stress interval and integrates by summing impacts over the full stress range for either stress increments or decrements. An alternate version, acct_base2012, can be used to calculate and incremental impacts for each stress interval, which can be assembled in Excel to illustrate solutions in terms of incremental and cumulative impacts over the ranges of the stresses, parameterized by the stress fraction from 0 to 1, i.e. from zero to 100 percent of total pumping.

Model run naming conventions

Naming conventions for the model runs used to calculate incremental impacts were invented for testing and demonstration purposes; they deserve explanation and, eventually, improvement. Model runs were initially named to calculate impacts of stress decrements with IWS in the base case. Additional model runs were prepared and named to calculate impacts of stress increments. These naming conventions are explained as follows.

Model runs for impact accounting with IWS in the base case are named as variants of the RRCA base cases (12p.* for years 1918-2000, and 2001-2010.* for years 2001-2010). Model runs for impact accounting without IWS in the base case are named as variants of the RRCA no-mound impact cases (12p4.* for years 1918-2000, and 2001-2010d.* for years 2001-2010).

Fractions are denoted "ptf". Examples: 0.0 is pt0, 0.025 is pt025, 0.1 is pt1, 0.5 is pt5 and 0.9 is pt9; but 1 ("ON") is denoted by 1. Model runs with all stresses held at the same fraction are denoted by a suffix "ptfALL", e.g. pt0ALL, pt025ALL, pt1ALL, pt5ALL and pt9ALL. Such model runs represent either reference or combined impact cases. With IWS in the base case, file names for a model run with all stress fractions at 50 pct have the suffix 12p _pt5ALL for the 12p model (1918-2000), and 2001-2010_pt5ALL for the 12s model (2001-2010). With no IWS in the base case, corresponding suffixes are 12p4 _pt5ALL and 2001-2010d_pt5ALL.

Stress decrements:

Model runs with a stress decrement applied to one of the three or four stresses are denoted by a suffix identifying the impact stress fraction followed by the source of the stress (CO, KS, NE or MD) and then the stress decrement; the reference stress fraction is not identified explicitly. The four possible sources of the stress correspond to pumping by each of the three States (CO, KS and NE) and Imported Water Supply (IWS), also identified as the mound and abbreviated MD for identifying model runs. Examples:

pt9CO_pt1: Colorado pumping is reduced by 10 percent from 100 to 90 percent. With IWS in the base case, suffixes for the 12p model (1918-2000) are 12p for the reference case and 12p_pt9CO_pt1 for the impact case. Suffixes for the 12s model beginning in 2001 2001-2010 for the reference case and and 2001-2010_pt9CO_pt1 for the impact case. With no IWS in the base case, corresponding suffixes are 12p4 and 12p4 _pt9CO_pt1 for the 12p model (1918-2000), and 2001-2010d and 2001-2010d_pt9CO_pt1 for the 12s model (2001-2010).

pt8CO_pt1: Colorado pumping is reduced by 10 pct from 90 pct to 80 pct. Reference and impact suffixes with IWS in the base case are 12p_pt9ALL and 12p_pt8CO_pt1 (12p model), and 2001-2010_pt9ALL and 2001-2010_pt8CO_pt1 (12s model). Reference and impact suffixes with no IWS in the base case are 12p4_pt9ALL and 12p4_pt8CO_pt1 (12p model), and 2001-2010d_pt9ALL and 2001-2010d_pt8CO_pt1 (12s model).

pt0CO_pt1: Colorado pumping is reduced by 10 pct from 10 pct to zero.

Stress increments:

Model runs with a stress increment applied to one of the three or four stresses are denoted by a suffix that identifies the reference stress fraction followed by the impact stress fraction and then the source of the stress (CO, KS, NE or MD); the stress increment is not identified explicitly. Examples:

pt8_pt9CO: Colorado pumping is increased by 10 pct from 80 pct to 90 pct. Reference and impact suffixes with IWS in the base case are 12p_pt8ALL and 12p_pt8_pt9CO (12p model), and 2001-2010_pt8ALL and 2001-2010_pt8_pt9CO (12s model). Reference and impact suffixes with no IWS in the base case would be 12p4_pt8ALL and 12p4_pt8_pt9CO (12p model), and 2001-2010d_pt8ALL and 2001-2010d_pt8_pt9CO (12s model).

pt9_1CO: Colorado pumping is increased by 10 pct from 90 pct to 100 pct. Reference and impact suffixes with IWS in the base case are 12p_pt9ALL and 12p_pt9_1CO (12p model), and 2001-2010_pt9ALL and 2001-2010_pt9_1CO (12s model). Reference and impact suffixes with no IWS in the base case would be 12p4_pt9ALL and 12p4_pt9_1CO (12p model), and 2001-2010d_pt9ALL and 2001-2010d_pt9_1CO (12s model).

pt0_pt1CO: Colorado pumping is increased by 10 pct from zero to 10 percent.

Application to ten-percent stress intervals with central differences

We show how the integrated solution is applied for ten-percent stress intervals with IWS excluded from the base case and central difference approximation of response functions.

Impacts of ten-percent pumping decrements begin from all pumping ON to 90 percent, with impacts calculated with respect to all pumping ON; then from 90 percent to 80 percent, with impacts calculated with respect to all pumping at 90 percent; and so on. The last ten-percent reduction is from ten percent to zero pumping, with impacts calculated with respect to all pumping at 10 percent.

Similarly, impacts of ten-percent pumping increments begin from all pumping OFF to 10 percent, with impacts calculated with respect to all pumping OFF; then from 10 percent to 20 percent, with impacts calculated with respect to all pumping at 10 percent; and so on. The last ten-percent increment is from 90 percent to 100 percent pumping, with impacts calculated with respect to all pumping at 90 percent.

Two forward difference approximations of pumping impacts are given by (a) the sum over the impacts of 10-percent pumping decrements, and (b) the sum over impacts of 10-percent pumping increments. The central difference approximation is given by the average of these two sums according to Equation (13). This is equivalent to taking the average of impacts due to 10-percent pumping decrements and increments, respectively, for each interval and summing over the intervals according to Equation (12). This approach is also useful for the purpose of showing accurate plots of incremental and cumulative impacts over the range of pumping. Some of these are shown below for future average conditions.

Results

The total impact discrepancy for the integrated solution is given by the difference between the sum of terms in Equation (5) and the total impact according to Equation (4). The discrepancy for the above example is plotted in Fig. 1 for years 1950-2010. Table 1 summarizes statistics for this discrepancy for the periods 1950-2000 (mean error = –0.8 afy, standard deviation = 4.0 afy). and 2001-2010 (mean error = –2.5 afy, standard deviation = 6.1 afy).

These statistics show that, within the error tolerance summarized in Table 1, the integrated solution equals the exact solution for the states’ gw CBCU. The exact solution cannot be calculated, since that would require continuous integration of the total differential (Equation (1); but we can come as close to it as we wish, limited only by computing requirements.

Table 2 summarizes the integrated solution for the states’ CBCU with no IWS in the base case, averaged over years 2001-2010 for each accounting point. The solution is calculated for 10-percent pumping intervals with centered response functions, i.e. central difference approximation of the partial derivatives. Columns (left to right) correspond to the accounting subbasins, computed baseflow without IWS, total impact of gw CBCU, each state’s gw CBCU, the IWS Credit according to the RRCA AP, net NE impact (gw CBCU – IWS Credit), sum of the states’ gw CBCU, and discrepancy (mean error and standard deviation for 2001-2010).

The two columns on the right-hand side of Table 2 show that this integrated solution has a negligible discrepancy over this time period for all accounting points. With a discrepancy this small, the integrated solution with 10-percent intervals and centered response functions effectively equals the exact solution that would be given by a continuous integration of the total differential.

Incremental and cumulative impacts: depletion characteristics for average futures

Each term of Equation (1) is a partial derivative (i.e. response function) integrated over a stress interval. Plots of incremental impacts illustrate the nonlinear behavior of depletion response over the range of pumping. The depletion response varies among subbasins and from year to year; and normally declines with increasing stress fraction. Cumulative plots, i.e. incremental impacts accumulated from zero to full pumping, illustrate the process of integrating the incremental impacts.

Plots of incremental and cumulative impacts were produced in an Excel file for a solution for average future conditions for years 2009-2059 with IWS excluded from the base case, approximated with response-centered, 10-pct intervals. These figures give an overview how stream depletion varies with pumping, but some subbasins show more extreme nonlinearity, such as Beaver and Sappa Creeks, and the Main Stem Swanson to Harlan County Lake.

Figs. 2a-c are plots for the total basin for future year 2059. Figs. 2a and 2b are line and bar graphs, respectively; both plot impacts in ten-percent increments of incremental impacts, and are simply two ways of showing the same data. Fig. 2a shows that the increments of total impact range in descending magnitude from 44.4 KAF to 12.4 KAF. The cumulative total impact in Fig. 32 is 290.3 KAF. The plots show indiscernible discrepancy between the sum of the states' gw CBCU and the total impact, as do plots of individual subbasins (Figs. 3-8).

Projected incremental and cumulative impacts in 2059 are shown for Beaver Creek (Figs. 3a-b), Sappa Creek (Figs. 4a-b), Main Stem Swanson-Harlan County Lake (Figs. 5a-b), Main Stem Above Swanson (Figs. 6a-b), Frenchman Basin (Figs. 7a-b) and South Fork (Figs. 8a-b).

For Beaver Creek in 2059, Fig. 3a shows that no baseflow is available for depletion above 70 percent total pumping, so the cumulative impact of pumping (Fig. 3b) is constant from 70 to 100 percent pumping.

Figs. 4a and 4b show that Sappa Creek baseflow is also completely depleted above 70 percent pumping. The incremental plots for Beaver and Sappa (Figs. 3a and 4a) both show how transitions over the range in pumping in how gw CBCU is distributed between KS and NE.

Incremental impacts for the Main Stem reach from Swanson to Harlan (Fig. 5a) make a transition from large positive impacts to negative, from nearly 8 KAF in the first 10 pct of pumping, a steady decline through 60 pct pumping, then a sharper decline for 70 pct pumping and negative impacts of –3 KAF for 80 pct pumping and –2 KAF for 90 pct and 100 pct pumping. Consequently, the cumulative impacts (Fig. 5b) rise to 38 KAF at 70 pct pumping and then fall to 31 KAF at 100 pct pumping.

The Main Stem reach above Swanson (Figs. 6a-b) also shows a transition for total impact increments from positive to slightly negative above 80 pct pumping (Fig. 6a), for which the magnitude of Colorado’s negative impact increments slightly exceed Nebraska’s positive impact increments.

Figs. 7a-b show projected impacts in 2059 for Frenchman Creek. Colorado’s incremental impacts shown in Fig. 7a decline over the range of pumping from 1625 af for the 0-10 pct pumping interval to 40 af for the 90-100 pct pumping interval. Colorado’s cumulative impact shown in Fig. 7b is 7360 af at 100 pct pumping, which is 8.3 percent of the total impact of 88.4 KAF.

Figs. 8a-b show projected impacts in 2059 for South Fork Republican River, based on the scenario that Bonny is included in the model. Incremental impacts decline steadily and then sharply for the 60-70 percent interval due to the Colorado CBCU component, which declines from 1675 af for the 50-60 pct interval to 175 af for the 60-70 pct interval. Kansas and Nebraska incremental impacts are nearly constant over the full range of pumping (Fig. 8a).

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Figures and Tables

Figures

Fig. 1. Total discrepancy for cumulative impact between 0 and 100 percent stress fraction.

Fig. 2a. Incremental impacts, total basin for average conditions projected to 2059.

Fig. 2b. Incremental impacts, total basin for average conditions projected to 2059.

Fig. 2c. Cumulative impacts, total basin for average conditions projected to 2059.

Fig. 3a. Beaver Creek: Incremental impacts projected to 2059.

Fig. 3b. Beaver Creek: Cumulative impacts projected to 2059.

Fig. 4a. Sappa Creek: Incremental impacts projected to 2059.

Fig. 4b. Sappa Creek: Cumulative impacts projected to 2059.

Fig. 5a. Main Stem, Swanson to Harlan County Lake: Incremental impacts projected to 2059.

Fig. 5b. Main Stem, Swanson to Harlan County Lake: Cumulative impacts projected to 2059.

Fig. 6a. Main Stem, above Swanson: Incremental impacts projected to 2059.

Fig. 6b. Main Stem, above Swanson: Cumulative impacts projected to 2059.

Fig. 7a. Frenchman Creek: Incremental impacts projected to 2059.

Fig. 7b. Frenchman Creek: Cumulative impacts projected to 2059.

Fig. 8a. South Fork Republican River: Incremental impacts projected to 2059.

Fig. 8b. South Fork Republican River: Cumulative impacts projected to 2059.

Tables

Table 1. Summary statistics for total impact discrepancy of integrated solution (ac-ft/yr).

Table 2. Summary of solution average for 2001-2010 with No IWS in base case, calculated for 10-percent pumping intervals, centered response functions: computed beneficial consumptive use (CBCU), IWS Credit and discrepancy. (Solution `sum_pt1avg_NoMD`)

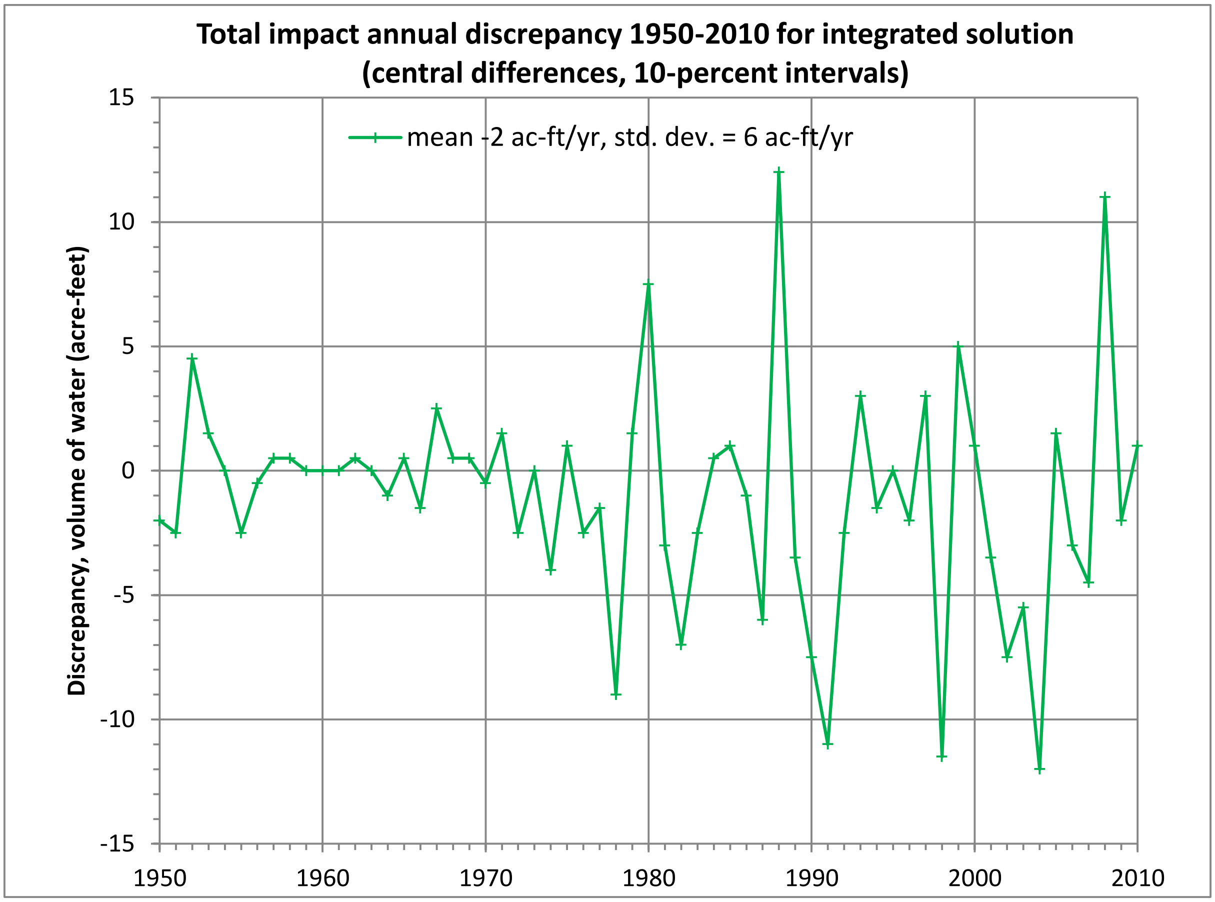

Fig. 1. Total discrepancy for cumulative impact between 0 and 100 percent stress fraction.

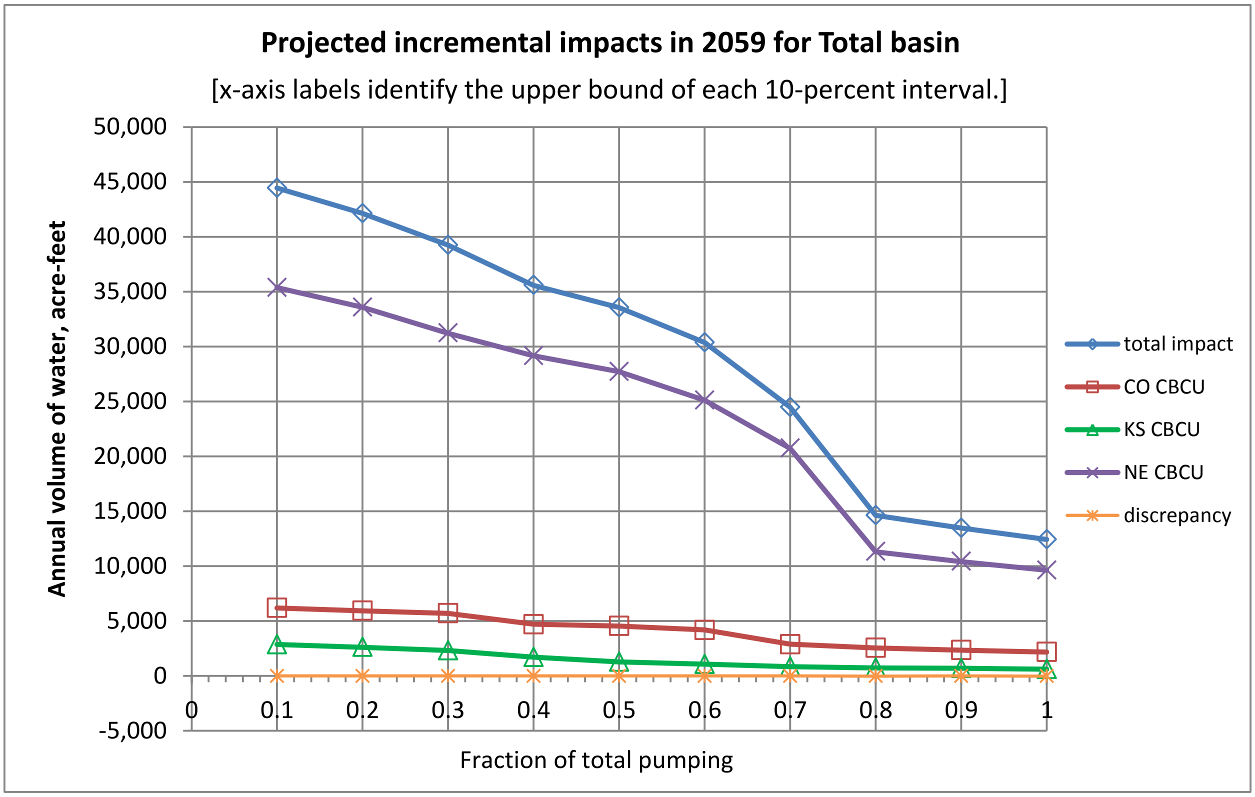

Fig. 2a. Incremental impacts, total basin for average conditions projected to 2059.

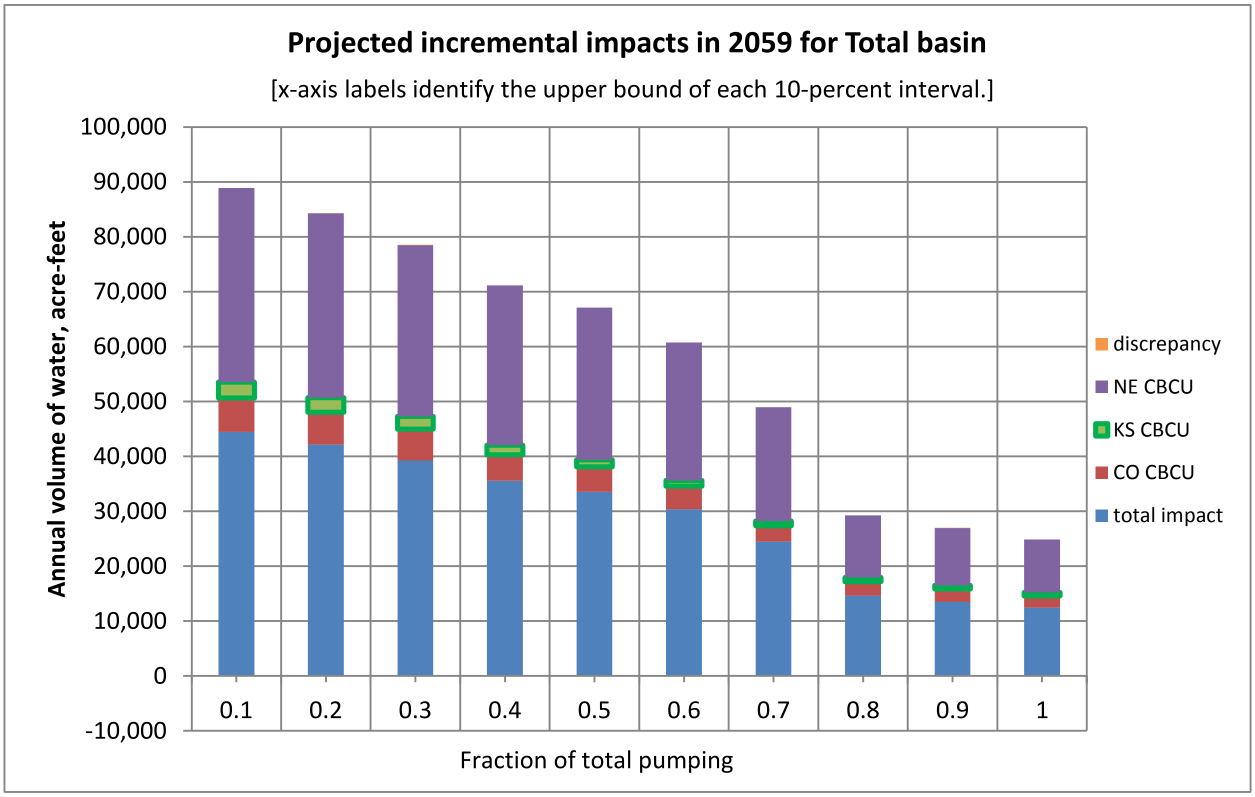

Fig. 2b. Incremental impacts, total basin for average conditions projected to 2059.

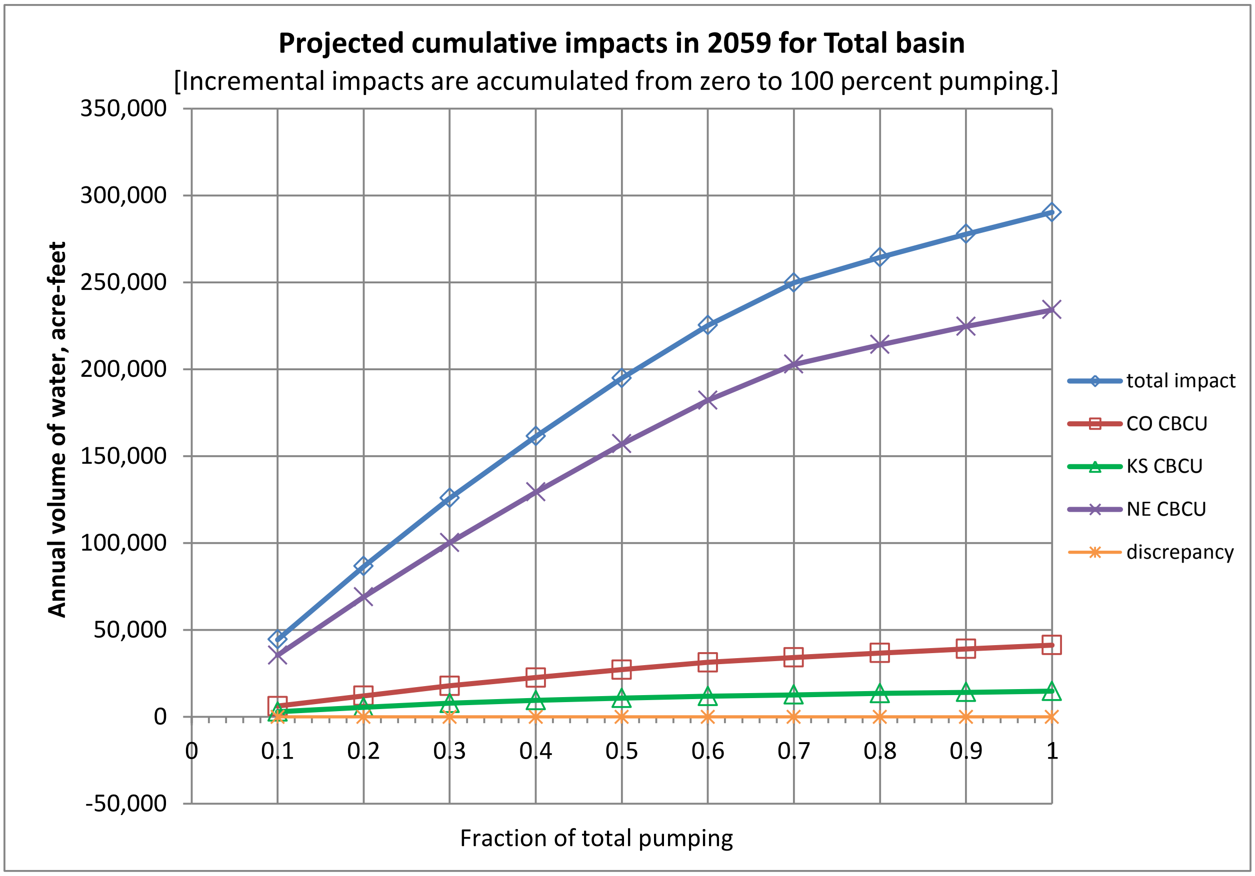

Fig. 2c. Cumulative impacts, total basin for average conditions projected to 2059.

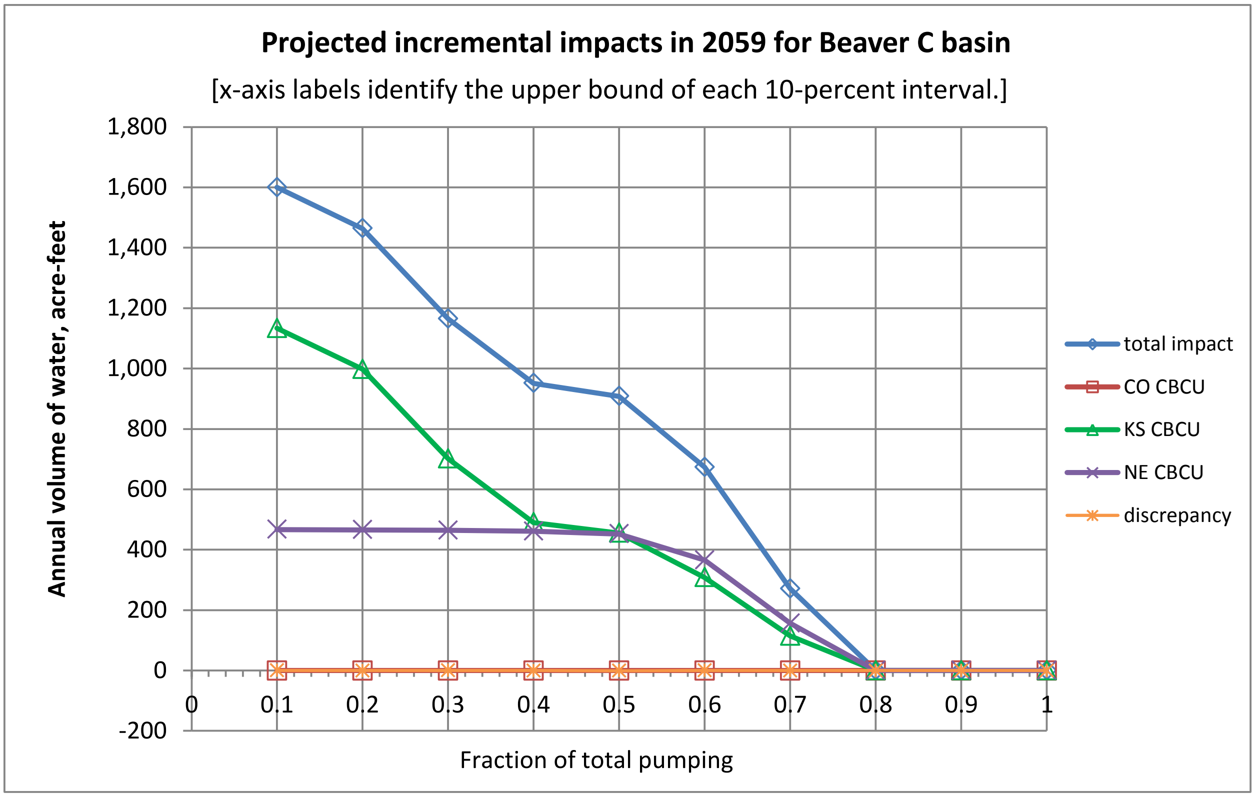

Fig. 3a. Beaver Creek: Incremental impacts projected to 2059.

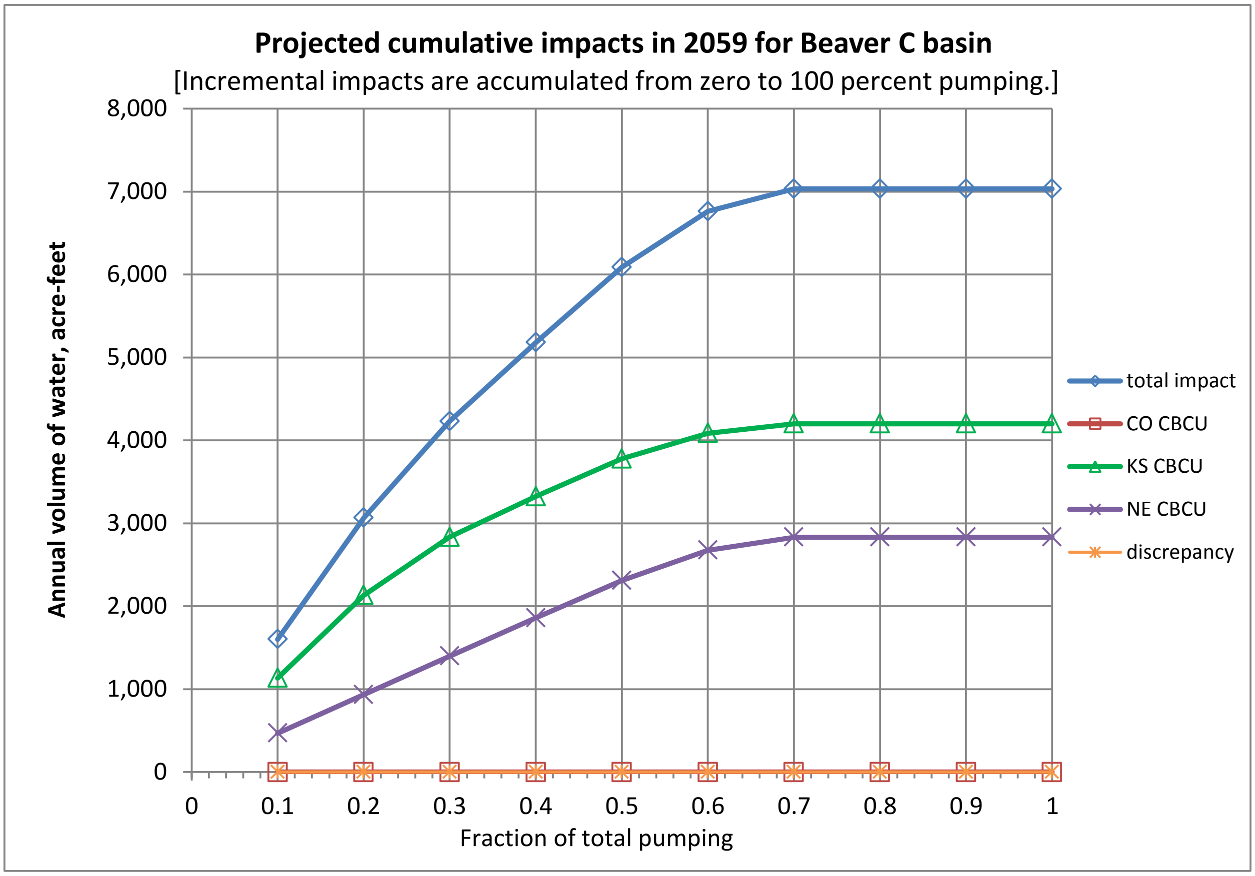

Fig. 3b. Beaver Creek: Cumulative impacts projected to 2059.

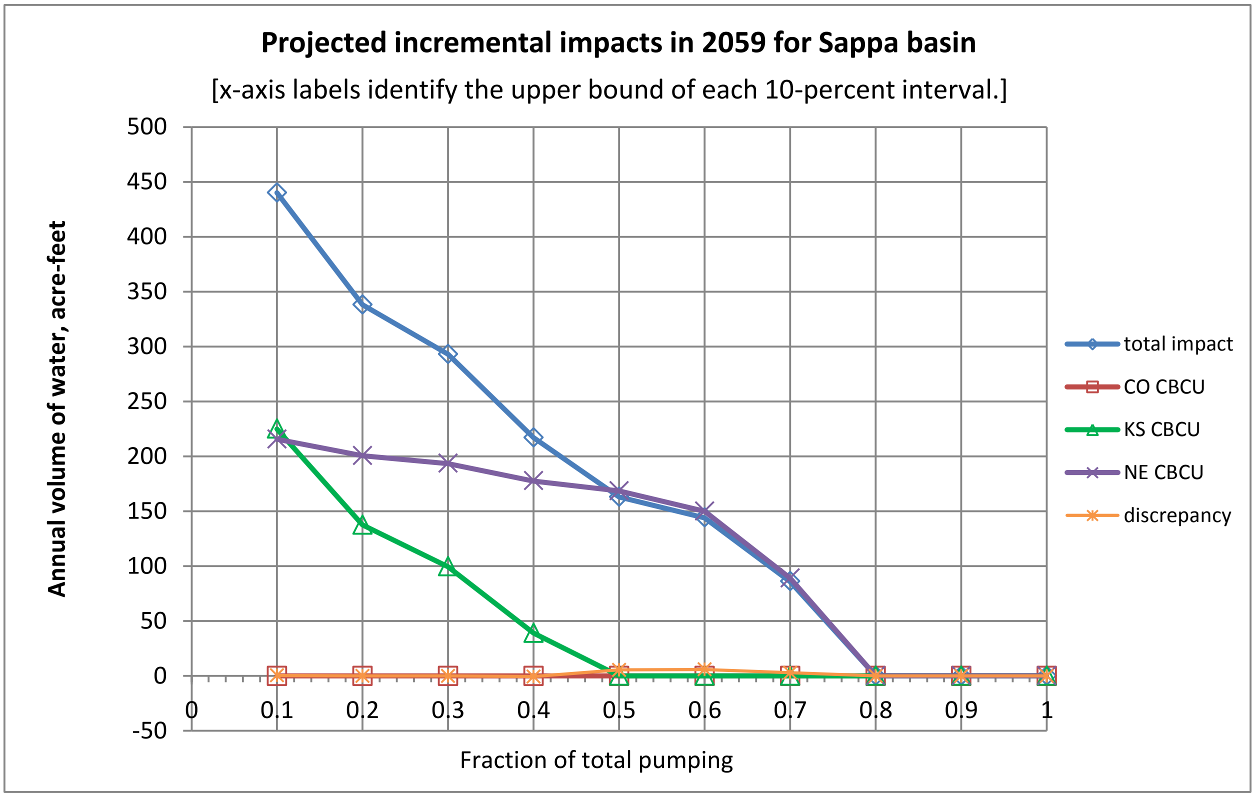

Fig. 4a. Sappa Creek: Incremental impacts projected to 2059.

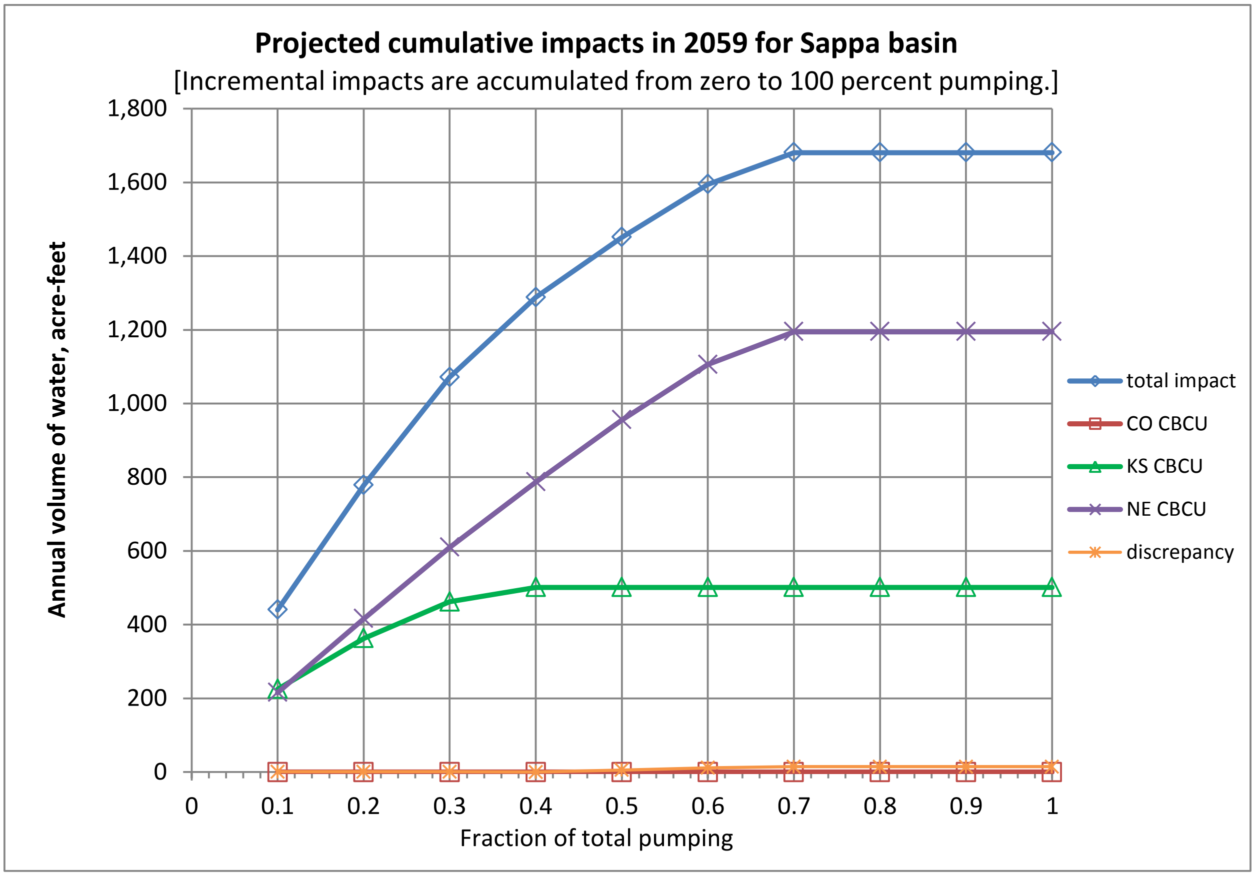

Fig. 4b. Sappa Creek: Cumulative impacts projected to 2059.

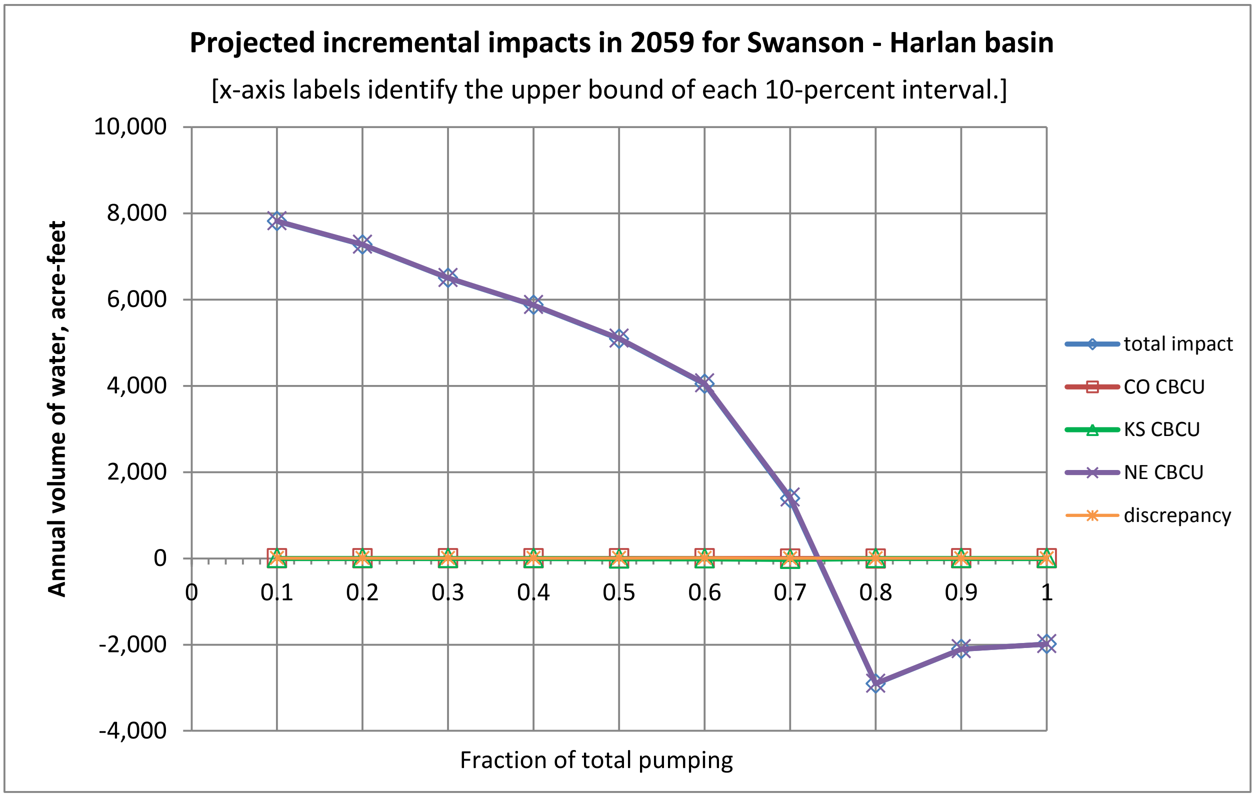

Fig. 5a. Main Stem, Swanson to Harlan County Lake: Incremental impacts projected to 2059.

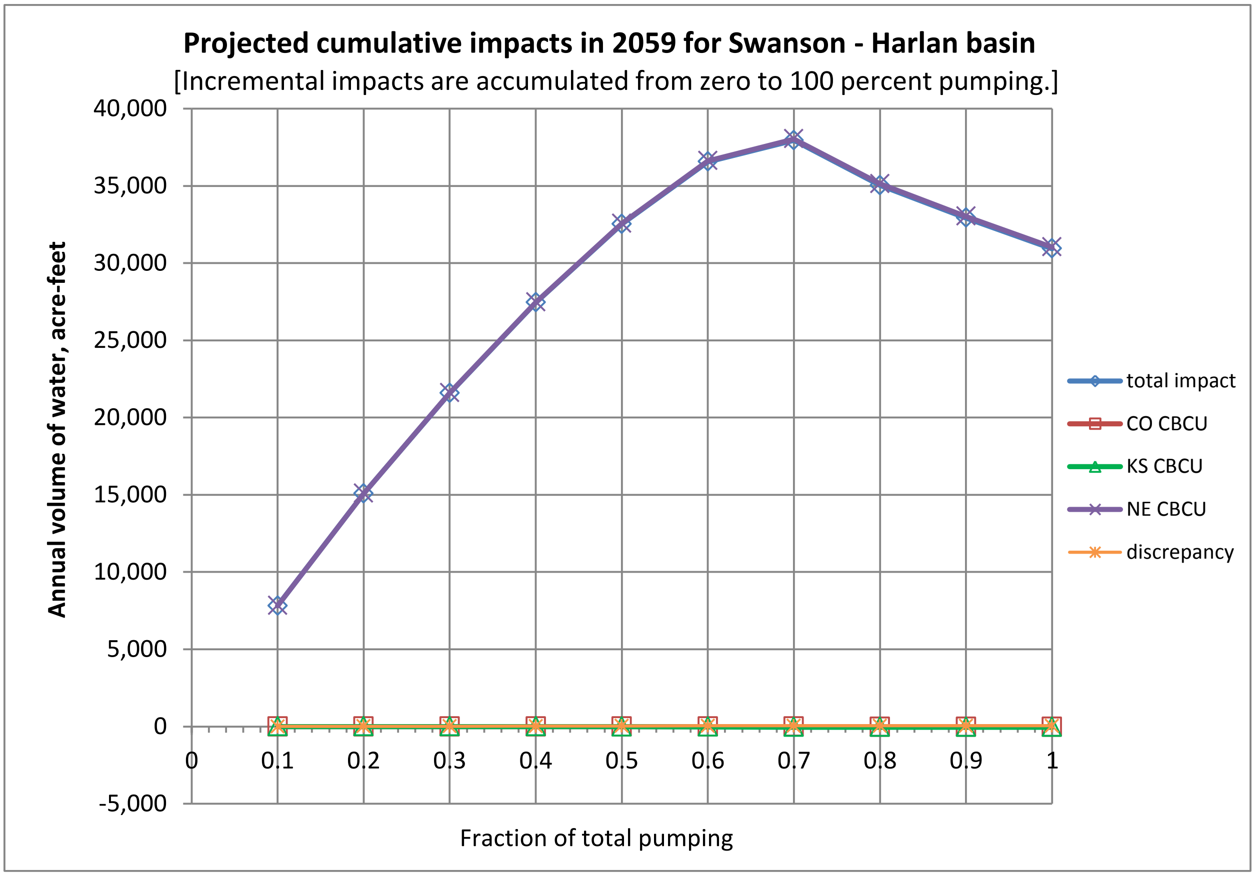

Fig. 5b. Main Stem, Swanson to Harlan County Lake: Cumulative impacts projected to 2059.

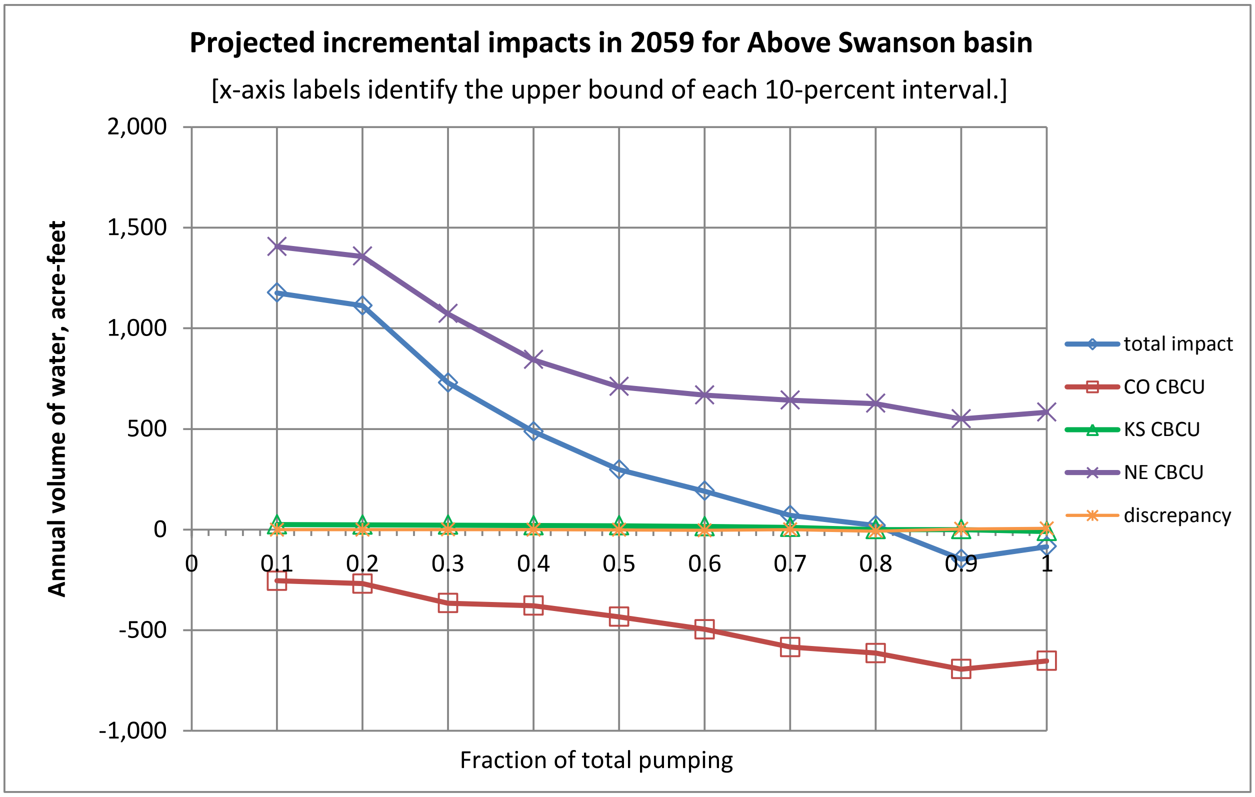

Fig. 6a. Main Stem, above Swanson: Incremental impacts projected to 2059.

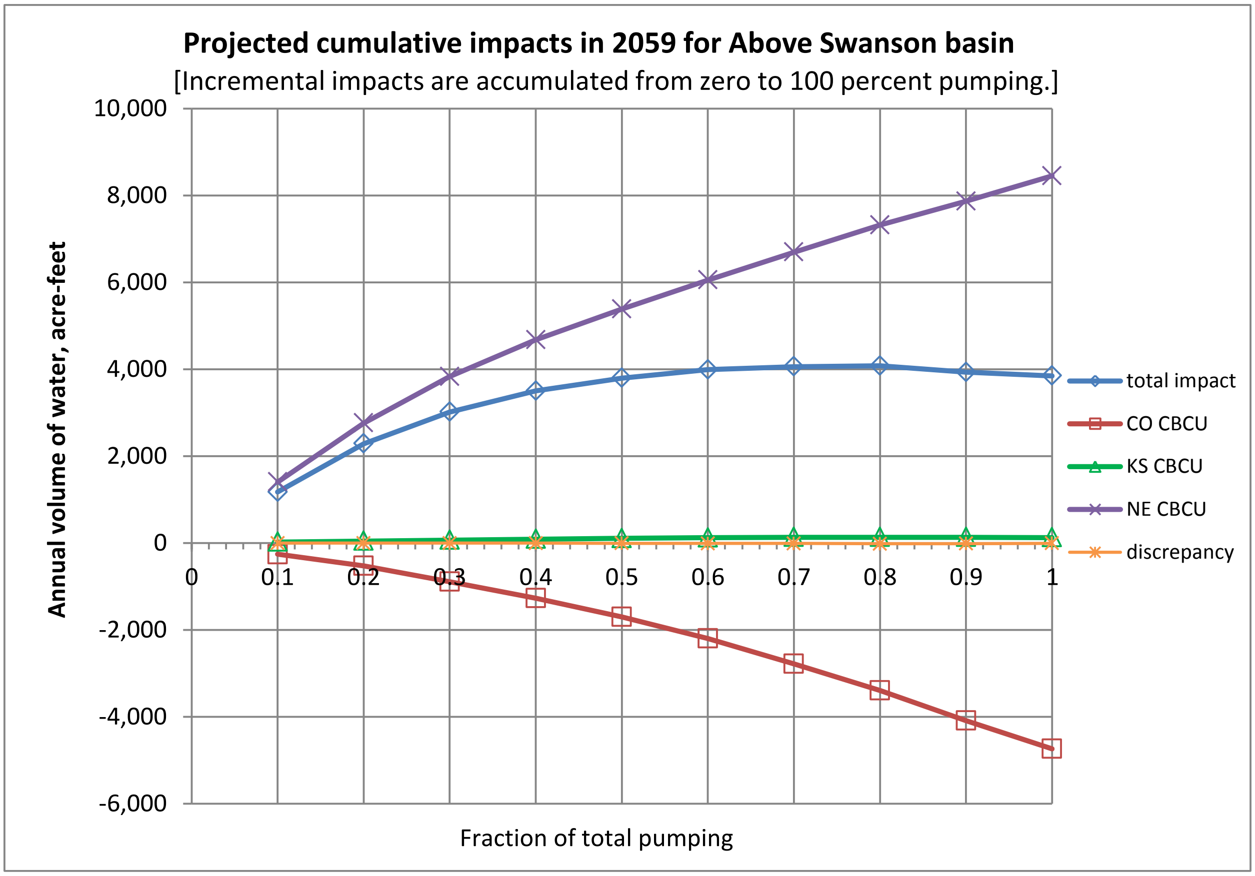

Fig. 6b. Main Stem, above Swanson: Cumulative impacts projected to 2059.

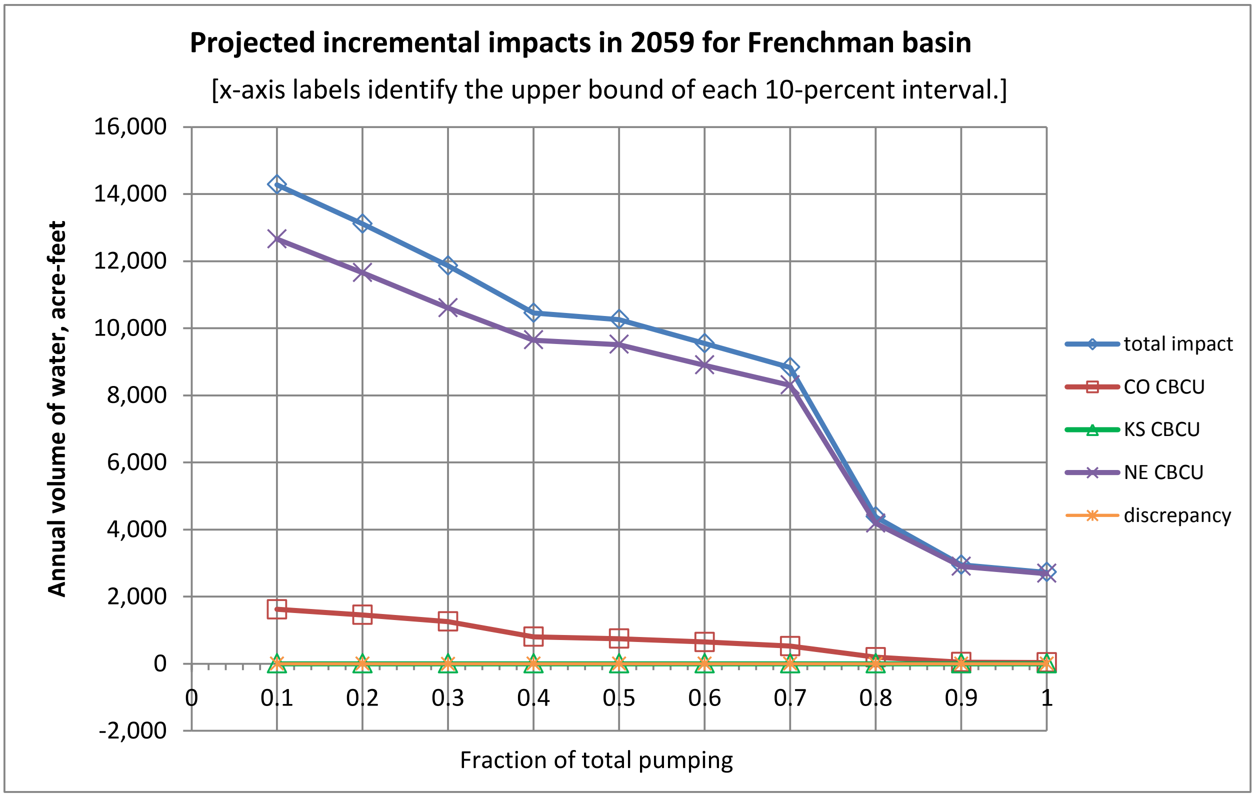

Fig. 7a. Frenchman Creek: Incremental impacts projected to 2059.

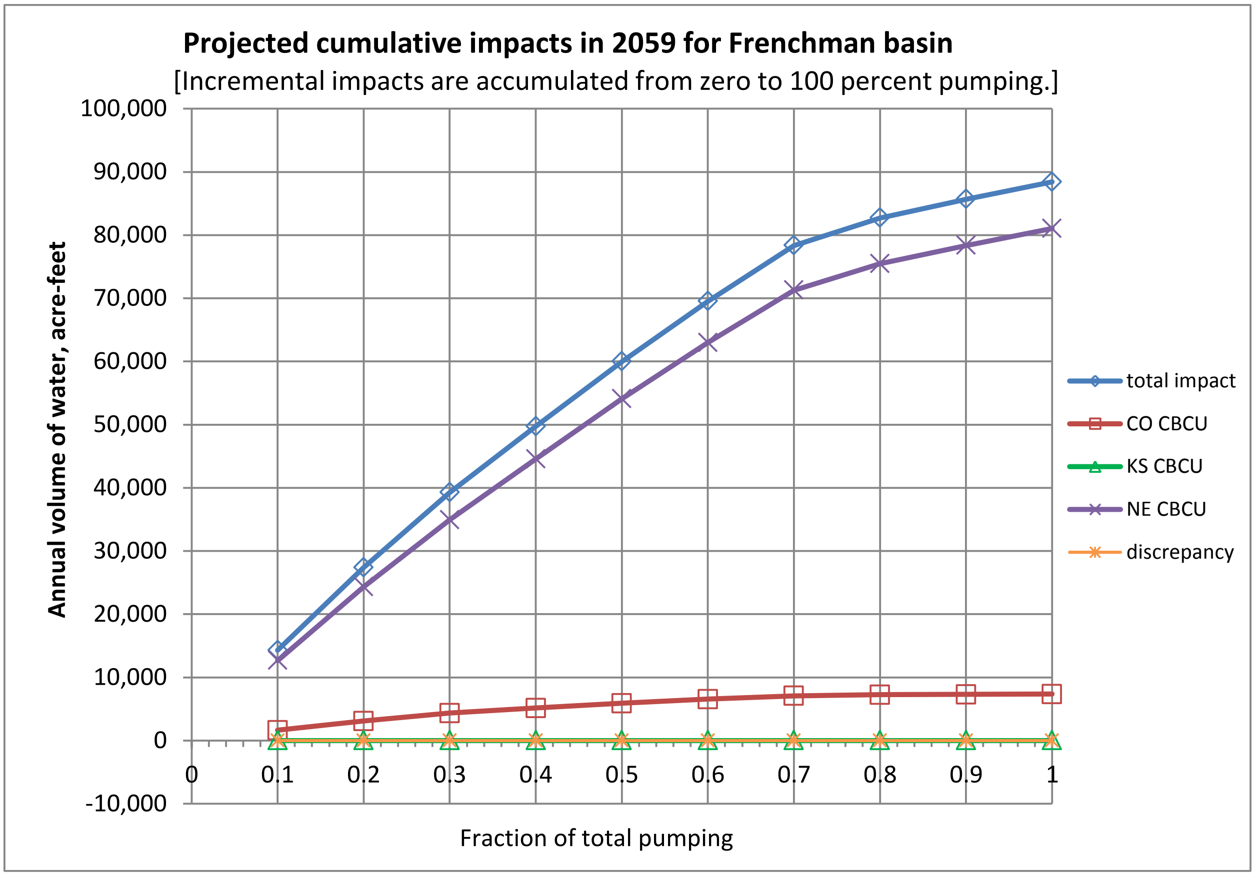

Fig. 7b. Frenchman Creek: Cumulative impacts projected to 2059.

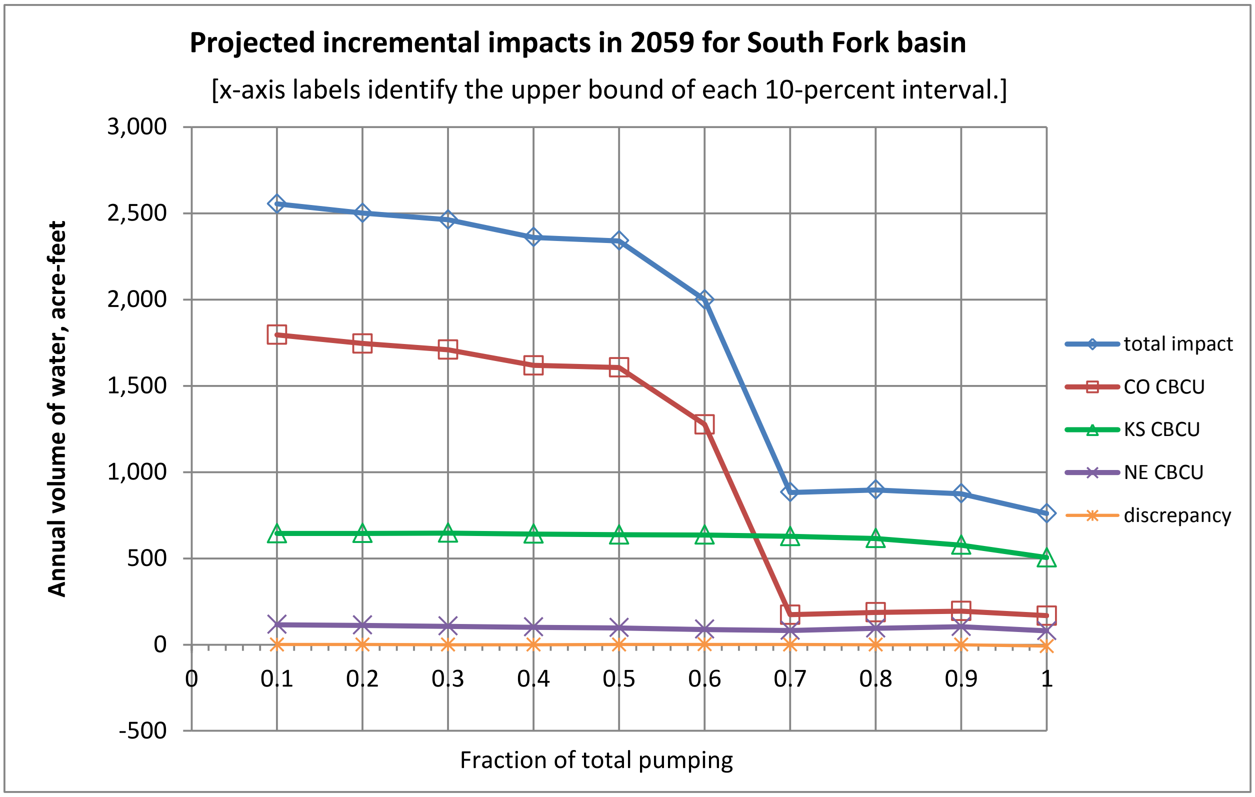

Fig. 8a. South Fork Republican River: Incremental impacts projected to 2059.

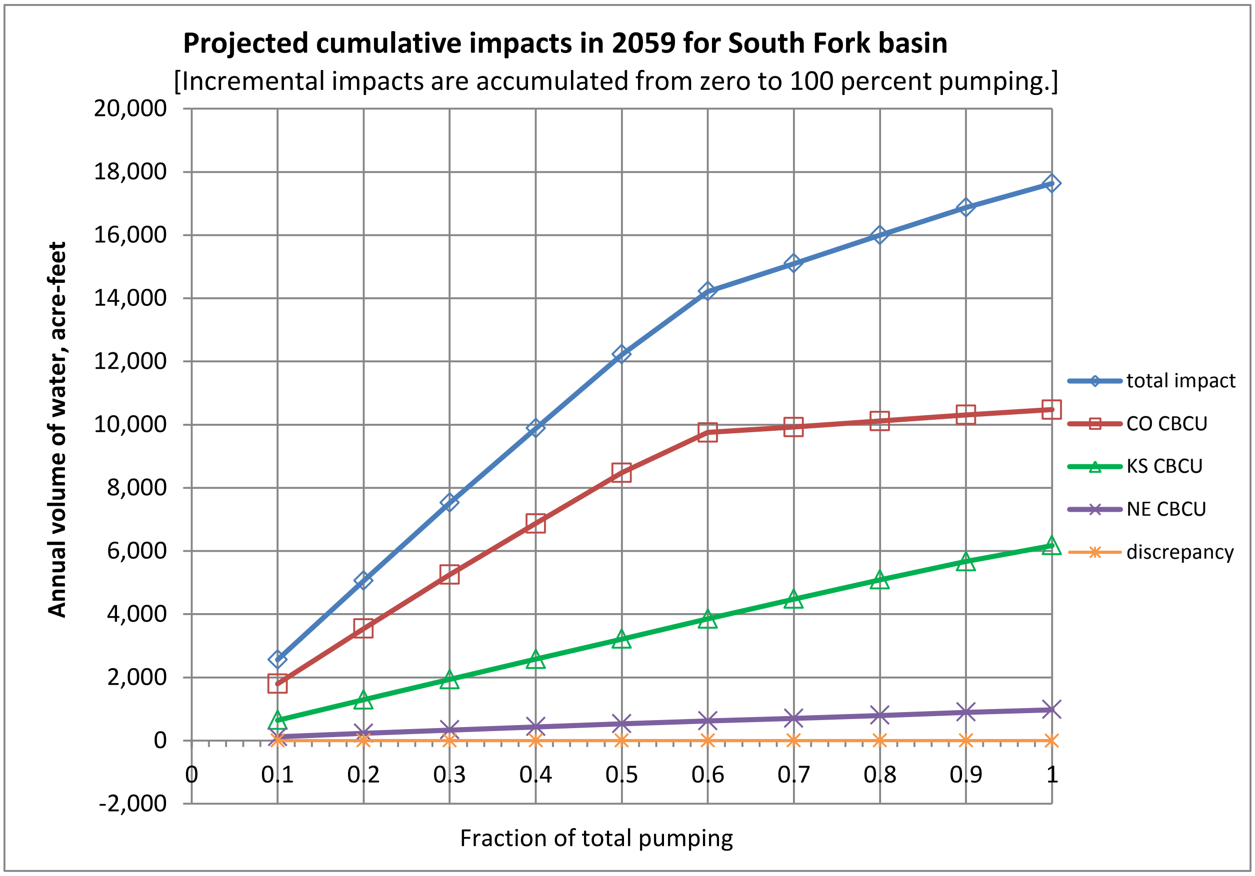

Fig. 8b. South Fork Republican River: Cumulative impacts projected to 2059.

Table 1. Summary statistics for total impact discrepancy of integrated solution (ac-ft/yr).

Period	mean	std dev	min	max
1950-2000	-0.8	4.0	-11.5	12.0
2001-2010	-2.5	6.1	-12.0	11.0

Table 2. Summary of solution average for 2001-2010 with No IWS in base case, calculated for 10-percent pumping intervals, centered response functions: computed beneficial consumptive use (CBCU), IWS Credit and discrepancy. (Solution sum_pt1avg_NoMD)

Account	computed baseflow (No IWS)	Total pumping impact	CO gw CBCU	KS gw CBCU	NE gw CBCU	IWS Credit	NE-IWS Credit	sum CO+KS+NE gw CBCU	mean error	std dev error
Arikaree	1027	1632	1249	123	260	0	260	1632	0	2
Beaver	691	9687	0	5098	4589	0	4589	9687	0	0
Buffalo	2176	3780	357	0	3423	0	3423	3780	0	0
Driftwood	1227	1324	0	0	1324	0	1324	1324	0	0
Frenchman	16481	83140	3170	0	79970	0	79970	83140	0	1
North Fork	32730	15401	14563	0	836	0	836	15399	-2	1
Above Swanson	-20422	10219	-2098	204	12112	0	12112	10219	0	5
Swanson - Harlan	-10832	29827	0	-21	29851	7118	22733	29830	3	2
Harlan - Guide Rock	37547	25465	0	0	25461	244	25217	25461	-4	4
Guide Rock - Hardy	15925	2893	0	57	2837	-1	2838	2893	0	0
Medicine	22725	20167	0	0	20167	9780	10387	20167	0	0
Prairie Dog	529	5253	0	5243	9	0	9	5251	-1	2
Red Willow	5115	6685	0	0	6685	36	6649	6685	0	0
Rock	5121	3889	71	0	3817	0	3817	3888	0	0
Sappa	237	3246	0	1196	2050	14	2035	3246	0	0
South Fork	4584	18678	11653	5786	1235	0	1235	18674	-4	8
Hugh Butler	513	1696	0	0	1696	0	1696	1696	0	0
Bonny	-1925	1269	1263	0	0	0	0	1263	-6	2
Keith Sebelius	2258	492	0	492	0	0	0	492	0	0
Enders	-3253	4468	0	0	4464	0	4464	4464	-4	1
Harlan	1574	871	0	60	811	16	796	871	0	0
Harry Strunk	-2555	347	0	0	347	0	347	347	0	0
Swanson	-1980	353	9	0	340	0	340	350	-4	3
Mainstem Total	22218	68404	-2101	243	70261	7356	62905	68404	1	4
Total	109492	250779	30241	18250	202286	17211	185075	250776	-2	6